4.1 Design of Reinforced Concrete Masonry Structures

Users Guide to NZS 4230:2004

Contents

Acknowledgement

This user guide was written by Jason Ingham and Kok Choon Voon, Department of Civil and Environmental Engineering, University of Auckland.

The authors wish to acknowledge the role of Standards New Zealand and of the committee members responsible for drafting NZS 4230:2004.

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Dr Peter Laursen and Dr Gavin Wight are thanked for their significant contributions pertaining to the design of unbonded post-tensioned masonry walls.

It is acknowledged that the contents of this user guide, and in particular the design examples, are derived or adapted from earlier versions, and the efforts of Emeritus Professor Nigel Priestley in formulating those design examples is recognised. It is acknowledged that the strut-and-tie model in section 3.8 is an adaption of that reported in Paulay and Priestley (1992).

The user guide was reviewed and revised by Dr Jason Ingham in 2012.

1.0 Introduction

NZS 4230 is the materials standard specifying the design and detailing requirements for masonry structures. The current version of this document has the full title “**NZS 4230:2004 Design of Reinforced Concrete Masonry Structures**”. The purpose of this user guide is to provide additional information explaining the rationale for new or altered clauses within this version of the Standard with respect to its predecessor versions, and to demonstrate the procedure in which it is intended that NZS 4230:2004 be used.

1.1 Background

The New Zealand masonry design standard was first introduced in 1985 as a provisional Standard NZS 4230P:1985. This document superseded NZS 1900 Chapter 9.2, and closely followed the format of NZS 3101 “Code of practice for the design of concrete structures”. The document was formally introduced in 1990 as NZS 4230:1990.

Since 1985 NZS 4230 was subject to significant amendment, firstly as a result of the publication of the revised loadings standard, NZS 4203:1992. This latter document contained major revisions to the formatting of seismic loadings, which typically are the structural design actions that dominate the design of most New Zealand concrete masonry structures. NZS 4203:1992 was itself revised with the introduction of the joint loadings standard AS/NZS 1170, with the seismic design criteria for New Zealand presented in part 5 or NZS 1170.5.

1.2 Related Standards

Whilst a variety of Standards are referred to within NZS 4230:2004, several documents merit special attention:

- As noted above, NZS 4230:2004 is the material design standard for reinforced concrete masonry, and is to be used in conjunction with the appropriate loadings standard defining the magnitude of design actions and loading combinations to be used in design. Unfortunately, release of NZS 1170.5 encountered significant delay, such that NZS 4230:2004 was released before NZS 1170.5 was available. The timing of these release dates led to Amendment No. 1 to NZS 4230:2004 being issued in December 2006 to ensure consistency with AS/NZS 1170 and NZS 1170.5.

- NZS 4230:2004 is to be used in the design of concrete masonry structures. The relevant document stipulating appropriate masonry materials and construction practice is NZS 4210:2001 “Masonry construction: Materials and workmanship”.

- NZS 4230:2004 is a specific design standard. Where the structural form falls within the scope of NZS 4229:1999 “Concrete Masonry Buildings Not Requiring Specific Engineering Design”, this latter document may be used as a substitute for NZS 4230:2004.

- NZS 4230:2004 is to be used in the design of concrete masonry structures. Its general form is intended to facilitate consultation with NZS 3101 “The design of concrete structures” standard, particularly for situations that are not satisfactorily considered in NZS 4230, but where engineering judgement may permit the content of NZS 3101 to indicate an appropriate solution.

2.0 Design Notes

The purpose of this chapter is to record and detail aspects of the Standard that differ from the previous version, NZS 4230:1990. While it is expected that the notes provided here will not address all potential queries, it is hoped that they may provide significant benefit in explaining the most significant changes presented in the latest release of the document.

2.1 Change of Title and Scope

The previous version of this document was titled “**NZS 4230:1990 Code of Practice for the Design of Masonry Structures**”. The new document has three separate changes within the title:

- The word **Code** has ceased to be used in conjunction with Standards documents to more clearly delineate the distinction between the New Zealand Building Code (NZBC), and the Standards that are cited within the Code. NZS 4230:2004 is intended for citation in Verification Method B1/VM1 of the Approved Documents for NZBC Clause B1 "Structure".

- The previous document was effectively intended to be used primarily for the design of reinforced **concrete** masonry structures, but did not preclude its use in the design of other masonry materials, such as clay or stone. As the majority of structural masonry constructed in New Zealand uses hollow concrete masonry units, and because the research used to underpin the details within the Standard almost exclusively pertain to the use of concrete masonry, the title was altered to reflect this.

- Use of the word **reinforced** is intentional. Primarily because the majority of structural concrete masonry in New Zealand is critically designed to support seismic loads, the use of unreinforced concrete masonry is excluded by the Standard. The only permitted use of unreinforced masonry in New Zealand is as a veneer tied to a structural element. Design of masonry veneers is addressed in Appendix F of NZS 4230:2004, in NZS 4210:2001, in NZS 4229:1999 and also in NZS 3604:2011 'Timber Framed Structures'. Veneer design outside the scope of these standards is the subject of special design, though some assistance may be provided by referring to AS 3700 'Masonry Structures'.

2.2 Nature of Commentary

Much of the information in NZS 4230:1990 was a significant departure from that contained in both previous New Zealand masonry standards, and in the masonry codes and standards of other countries at that time. This was primarily due to the adoption of a limit state design approach, rather than the previous "allowable stress" method, and because the principle of capacity design had only recently been fully developed. Consequently, NZS 4230:1990: Part 2 contained comprehensive details on many aspects of structural seismic design that were equally applicable for construction using other structural materials.

Since release of NZS 4230:1990, much of the commentary details have been assembled within a text by Paulay and Priestley[1]. For NZS 4230:2004 it was decided to produce an abbreviated commentary that primarily addressed aspects of performance specific to concrete masonry.

This abbreviation permitted the Standard and the commentary to be produced as a single document, which was perceived to be preferable to providing the document in two parts. Consequently, designers may wish to consult the aforementioned text, or NZS 4230:1990:Part 2, if they wish to refresh themselves on aspects of general structural seismic design, such as the influence of structural form and geometry on seismic response, or the treatment of dynamic magnification to account for higher mode effects. In addition, care has been taken to avoid unnecessarily replicating information contained within NZS 3101, such that that Standard is in several places referred to in NZS 4230:2004.

2.3 Material Strengths

In the interval between release of NZS 4230:1990 and NZS 4230:2004 a significant volume of data has been collected pertaining to the material characteristics of concrete masonry. The availability of this new data has prompted the changes detailed below.

2.3.1 Compression Strength f'_m

The most significant change in material properties is that the previously recommended compressive strength value for Observation Type B masonry was found to be unduly conservative.

As identified in NZS 4210, the production of both concrete masonry units and of block-fill grout is governed by material standards. Accounting for the statistical relationship between the mean strength and the lower 5% characteristic strength for these constituent materials, it follows that a default value of $f'_m = 12$ MPa is appropriate for Observation Type B. This is supported by a large volume of masonry prism test results, and an example of the calculation conducted to establish this value is presented here in section 3.1.

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[1] Paulay, T., and Priestley, M. J. N. (1992) "Seismic Design of Reinforced Concrete and Masonry Buildings", John Wiley and Sons, New York, 768 pp.

2.3.2 Modulus of Elasticity of Masonry, E_m

As detailed in section 3.4.2 of NZS 4230:2004, the modulus of elasticity of masonry is to be taken as E_m = 15 GPa. This value is only 60% of the value adopted previously.

Discussion with committee members responsible for development of NZS 4230P:1985 has indicated that the previously prescribed value of E_m = 25 GPa was adopted so that it would result in conservatively large stiffness, resulting in reduced periods and therefore larger and more conservative seismic loads. However, the value of E_m = 25 GPa is inconsistent with both measured behaviour and with a widely recommended relationship for concrete masonry of $E_m \approx 1000 f'_m$, representing a secant stiffness passing through the point (f'_m, ε_m = 0.001) on the stress strain curve.

Note also that application of this equation to 3.4.2 captures the notion that f'_m (12 MPa) is the lower 5% characteristic strength but that E_m (15 GPa) is the mean modulus of elasticity. This relationship is quantitatively demonstrated here in section 3.1.

It is argued that whilst period calculation may warrant a conservatively high value of E_m, serviceability design for deformations merits a correspondingly low value of E_m to be adopted. Consequently, the value of E_m = 15 GPa is specified as a mean value, rather than as an upper or a lower characteristic value.

2.3.3 Ultimate Compression Strain, ε_u

NZS 4230:1990 specified an ultimate compression strain for unconfined concrete masonry of ε_u = 0.0025. This value was adopted somewhat arbitrarily in order to be conservatively less than the comparable value of ε_u = 0.003 which is specified in NZS 3101 for the design of concrete structures.

In the period since development of NZS 4230:1990 it has become accepted internationally, based upon a wealth of physical test results, that there is no evidence to support a value other than that adopted for concrete. Consequently, when using NZS 4230:2004 the ultimate compression strain of unconfined concrete masonry shall be taken as ε_u = 0.003.

2.3.4 Strength Reduction Factors

Selection of strength reduction factors should be based on comprehensive studies on the measured structural performance of elements when correlated against their predicted strength, in order to determine the effect of materials and of construction quality.

The strategy adopted in NZS 4230:1990 was to consider the values used in NZS 3101, but to then add additional conservatism based on the perception that masonry material strength characteristics and construction practices were less consistent than their reinforced concrete equivalent.

In NZS 4230:2004 the strength reduction factors have been altered with respect to their predecessors because:

1. The manufacture of masonry constituent materials and the construction of masonry structures are governed by the same regulatory regimes as those of reinforced concrete.

2. There is no measured data to form a basis for the adoption of values of the strength reduction factors other than those employed in NZS 3101 for concrete structures, and the adoption of corresponding values will facilitate designers interchanging between NZS 4230 and NZS 3101.

3. The values adopted in NZS 4230:2004 are more conservative than those prepared by the Masonry Standards Joint Committee[2] (comprised of representatives from The Masonry Society, the American Concrete Institute, and the American Society of Civil Engineers), were ū = 0.9 is specified for reinforced masonry in flexure and ū = 0.8 is specified for reinforced masonry in shear.

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[2] Masonry Standards Joint Committee (2011) ñBuilding Code Requirements for Masonry Structuresò and ñSpecification for Masonry Structuresò, TMS 402-11/ACI 530-11/ASCE 5-11, USA.

2.4 Design Philosophies

Table 3-2 of NZS 4230:2004 presents four permitted design philosophies, primarily based upon the permitted structural ductility factor, ε.

Whilst all design philosophies are equally valid, general discussion amongst designers of concrete masonry structures tends to suggest that nominally ductile and limited ductile response is most regularly favoured.

Taking due account for overall structural behaviour in order to avoid brittle failure mechanisms, nominally ductile design has the advantage over elastic design of producing reduced seismic design actions without requiring any special seismic detailing.

2.4.1 Limited Ductile Design

As outlined in section 3.7.3 of NZS 4230:2004, when conducting limited ductile design it is permitted to either adopt capacity design principles, or to use a simplified approach (3.7.3.3). In the simplified approach, where limits are placed on building height, the influence of material overstrength and dynamic magnification are accounted for by amplifying the seismic moments outside potential plastic hinge regions by an additional 50% (Eqn. 3-3) and by applying the seismic shear forces throughout the structure by an additional 100% (Eqn. 3-4). Consequently, the load combinations become:

$$\phi M_n \geq M_G^* + M_{Qu}^* + 1.5M_E^* \quad \text{and} \quad \phi V_n \geq V_G^* + V_{Qu}^* + 2V_E^* .$$

2.5 Component Design

An important modification to NZS 4230:2004 with respect to its predecessors is the use of a document format that collects the majority of criteria associated with specific components into separate sections.

This format is a departure from earlier versions which were formatted based upon design actions. The change was adopted because the new format was believed to be more helpful for users of the document.

The change also anticipated the release of NZS 3101:2006 to adopt a similar format, and is somewhat more consistent with equivalent Standards from other countries, particular AS 3700.

2.5.1 Definition of Column

Having determined that the design of walls, beams, and columns would be dealt with in separate sections, it was deemed important to clearly establish the distinction between a wall and a column.

In Section 2 of the standard it is stated that a column is an element having a length not greater than 790 mm and a width not less that 240 mm, subject primarily to compressive axial load. However, the intent of Section 7.3.1.5 was that a wall having a length less than 790 mm and having a compressive axial load less than $0.1f'_m A_g$ may be designed as either a wall or as a column depending on the intended function of the component within the design strategy, recognising that the design criteria for columns are more stringent than those for walls.

2.5.2 Moment Capacity of Walls

Moment capacity may be calculated from first principles using a linear distribution of strain across the section, the appropriate magnitude of ultimate compression strain, and the appropriate rectangular stress block. Alternatively, for **Rectangular**-section masonry components with **uniformly** distributed flexural reinforcement, Tables 2 to 5 over the page may be used.

These tables list in non-dimensional form the nominal capacity of unconfined and confined concrete masonry walls with either Grade 300 or Grade 500 flexural reinforcement, for different values of the two salient parameters, namely the axial load ratio $N_n/f'_m L_w t$ or $N_n/Kf'_m L_w t$, and the strength-adjusted reinforcement ratio pf_y/f'_m or pf_y/Kf'_m.

Charts, produced from Tables 2 to 5, are also plotted which enable the user to quickly obtain a value for pf_y/f'_m or pf_y/Kf'_m given the axial load ratio N_n/f'_mL_wt or N_n/Kf'_mL_wt and the moment ratio $M_n/f'_mL_w^2t$ or $M_n/Kf'_mL_w^2t$. These charts are shown as Figures 1 to 4.

On the charts, each curve represents a different value for pf_y/f'_m or pf_y/Kf'_m. For points which fall between the curves, values can be established using linear interpolation.

2.6 Maximum Bar Diameters

Whilst not changed from the values given in NZS 4230:1990, it is emphasised here that there are limits to the permitted bar diameter that may be used for different component types, as specified in 7.3.4.5, 8.3.6.1 and 9.3.5.1.

Furthermore, as detailed in C7.3.4.5 there are limits to the size of bar that may be lapped, which makes a more restrictive requirement when using grade 500 MPa reinforcement.

Consequently, the resulting maximum bar sizes are presented below.

Table 1: **Maximum bar diameter for different block sizes**

Block size (mm)	Walls and beams		Columns	
	f_y = 300 MPa	f_y = 500 MPa	f_y = 300 MPa	f_y = 500 MPa
140	D16	DH12	5-D10	3-DH10
190	D20	DH16	3-D16	DH16
240	D25	DH20	2-D20	DH20
390	---	---	D32	DH32

2.7 Ductility Considerations

The Standard notes in section 7.4.6 that unless confirmed by a special study, adequate ductility may be assumed when the neutral axis depth of a component is less than an appropriate fraction of the section depth. Section 2.7.1 below lists the ratios c/L_w for masonry walls while justification for the relationship limiting the neutral axis depth is presented in sections 2.7.2 and 3.4.

An outline of the procedure for conducting a special study to determine the available ductility of cantilevered concrete masonry walls is presented in section 2.7.3.

2.7.1 Neutral Axis Depth

Neutral axis depth may be calculated from first principles, using a linear distribution of strain across the section, the appropriate level of ultimate compression strain and the appropriate rectangular stress block.

Alternatively, for **Rectangular** section structural walls, Tables 6 and 7 may be used.

These tables list in non-dimensional form the neutral axis depth of unconfined and confined walls with either Grade 300 or Grade 500 flexural reinforcement, for different values of axial load ratio N_n/f'_mL_wt or N_n/Kf'_mL_wt and reinforcement ratio pf_y/f'_m or pf_y/Kf'_m, where p is the ratio of uniformly distributed vertical reinforcement.

Charts, produced from Tables 6 and 7, are also plotted which enable the user to quickly obtain a value for c/L_w given the axial load ratio N_n/f'_mL_wt or N_n/Kf'_mL_wt and different value of pf_y/f'_m or pf_y/Kf'_m. These charts are shown as Figures 5 and 6.

Table 2: $\frac{M_n}{f'_m L_w^2 t}$ **for unconfined wall with f_y = 300 MPa**

$\frac{pf_y}{f'_m}$	Axial Load Ratio $\frac{N_n}{f'_m L_w t}$								
	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0.00	0.000	0.0235	0.0441	0.0618	0.0765	0.0882	0.0971	0.1029	0.1059
0.01	0.0049	0.0279	0.0480	0.0652	0.0795	0.0909	0.0995	0.1052	0.1079
0.02	0.0097	0.0322	0.0518	0.0686	0.0826	0.0937	0.1020	0.1075	0.1102
0.04	0.0190	0.0406	0.0593	0.0753	0.0886	0.0992	0.1070	0.1122	0.1146
0.06	0.0280	0.0487	0.0665	0.0818	0.0945	0.1045	0.1120	0.1168	0.1190
0.08	0.0367	0.0566	0.0735	0.0881	0.1002	0.1099	0.1169	0.1215	0.1235
0.10	0.0451	0.0641	0.0804	0.0944	0.1059	0.1152	0.1218	0.1261	0.1279
0.12	0.0534	0.0713	0.0871	0.1005	0.1116	0.1204	0.1267	0.1307	0.1324
0.14	0.0613	0.0783	0.0936	0.1064	0.1171	0.1255	0.1315	0.1353	0.1369
0.16	0.0690	0.0853	0.0999	0.1123	0.1225	0.1306	0.1363	0.1399	0.1414
0.18	0.0762	0.0922	0.1062	0.1181	0.1279	0.1357	0.1411	0.1445	0.1459
0.20	0.0832	0.0989	0.1124	0.1238	0.1332	0.1406	0.1459	0.1491	0.1503

Table 3: $\frac{M_n}{f'_m L_w^2 t}$ **for unconfined wall with f_y = 500 MPa**

$\frac{pf_y}{f'_m}$	Axial Load Ratio $\frac{N_n}{f'_m L_w t}$								
	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0.00	0.000	0.0235	0.0441	0.0618	0.0765	0.0882	0.0971	0.1029	0.1059
0.01	0.0049	0.0279	0.0480	0.0652	0.0794	0.0908	0.0993	0.1049	0.1076
0.02	0.0097	0.0322	0.0517	0.0685	0.0824	0.0934	0.1015	0.1068	0.1093
0.04	0.0190	0.0405	0.0591	0.0750	0.0881	0.0984	0.1059	0.1107	0.1128
0.06	0.0280	0.0484	0.0662	0.0813	0.0937	0.1033	0.1103	0.1147	0.1163
0.08	0.0365	0.0561	0.0731	0.0874	0.0992	0.1081	0.1147	0.1186	0.1199
0.10	0.0448	0.0635	0.0797	0.0934	0.1043	0.1129	0.1190	0.1225	0.1234
0.12	0.0528	0.0707	0.0862	0.0992	0.1096	0.1176	0.1233	0.1264	0.1271
0.14	0.0605	0.0777	0.0925	0.1047	0.1147	0.1223	0.1275	0.1303	0.1307
0.16	0.0680	0.0844	0.0986	0.1103	0.1198	0.1269	0.1318	0.1342	0.1344
0.18	0.0752	0.0910	0.1045	0.1157	0.1247	0.1315	0.1359	0.1381	0.1380
0.20	0.0823	0.0974	0.1104	0.1211	0.1297	0.1359	0.1400	0.1420	0.1417

Table 4: $\frac{M_n}{Kf'_m L_w^2 t}$ **for confined wall with f_y = 300 MPa**

$\frac{pf_y}{Kf'_m}$	Axial Load Ratio $\frac{N_n}{Kf'_m L_w t}$								
	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0.00	0.000	0.0236	0.0444	0.0625	0.0778	0.0903	0.1000	0.1069	0.1111
0.01	0.0049	0.0280	0.0484	0.0661	0.0810	0.0933	0.1027	0.1095	0.1136
0.02	0.0098	0.0324	0.0523	0.0696	0.0842	0.0962	0.1055	0.1121	0.1161
0.04	0.0191	0.0409	0.0599	0.0766	0.0905	0.1020	0.1108	0.1173	0.1211
0.06	0.0281	0.0491	0.0673	0.0833	0.0967	0.1078	0.1163	0.1224	0.1261
0.08	0.0369	0.0569	0.0746	0.0899	0.1029	0.1135	0.1217	0.1275	0.1311
0.10	0.0454	0.0645	0.0818	0.0964	0.1089	0.1191	0.1271	0.1326	0.1360
0.12	0.0537	0.0720	0.0888	0.1027	0.1149	0.1246	0.1323	0.1377	0.1410
0.14	0.0616	0.0794	0.0956	0.1090	0.1209	0.1302	0.1376	0.1428	0.1459
0.16	0.0692	0.0867	0.1021	0.1152	0.1267	0.1357	0.1428	0.1479	0.1509
0.18	0.0767	0.0939	0.1085	0.1214	0.1324	0.1412	0.1480	0.1530	0.1558
0.20	0.0841	0.1009	0.1149	0.1275	0.1381	0.1466	0.1532	0.1581	0.1608

Table 5: $\frac{M_n}{Kf'_m L_w^2 t}$ **for confined wall with f_y = 500 MPa**

$\frac{pf_y}{Kf'_m}$	Axial Load Ratio $\frac{N_n}{Kf'_m L_w t}$								
	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0.00	0.000	0.0236	0.0444	0.0625	0.0778	0.0903	0.1000	0.1069	0.1111
0.01	0.0049	0.0280	0.0484	0.0661	0.0809	0.0932	0.1027	0.1094	0.1135
0.02	0.0098	0.0324	0.0523	0.0696	0.0841	0.0961	0.1054	0.1120	0.1159
0.04	0.0191	0.0408	0.0599	0.0765	0.0904	0.1019	0.1107	0.1171	0.1208
0.06	0.0281	0.0489	0.0673	0.0832	0.0967	0.1076	0.1161	0.1221	0.1257
0.08	0.0369	0.0569	0.0746	0.0898	0.1027	0.1133	0.1214	0.1272	0.1306
0.10	0.0454	0.0646	0.0817	0.0962	0.1088	0.1188	0.1267	0.1322	0.1355
0.12	0.0534	0.0720	0.0887	0.1026	0.1146	0.1243	0.1320	0.1372	0.1403
0.14	0.0614	0.0794	0.0956	0.1089	0.1205	0.1298	0.1372	0.1422	0.1452
0.16	0.0692	0.0866	0.1018	0.1151	0.1262	0.1352	0.1424	0.1472	0.1500
0.18	0.0769	0.0938	0.1083	0.1212	0.1319	0.1406	0.1475	0.1522	0.1549
0.20	0.0843	0.1006	0.1148	0.1273	0.1377	0.1460	0.1527	0.1573	0.1598

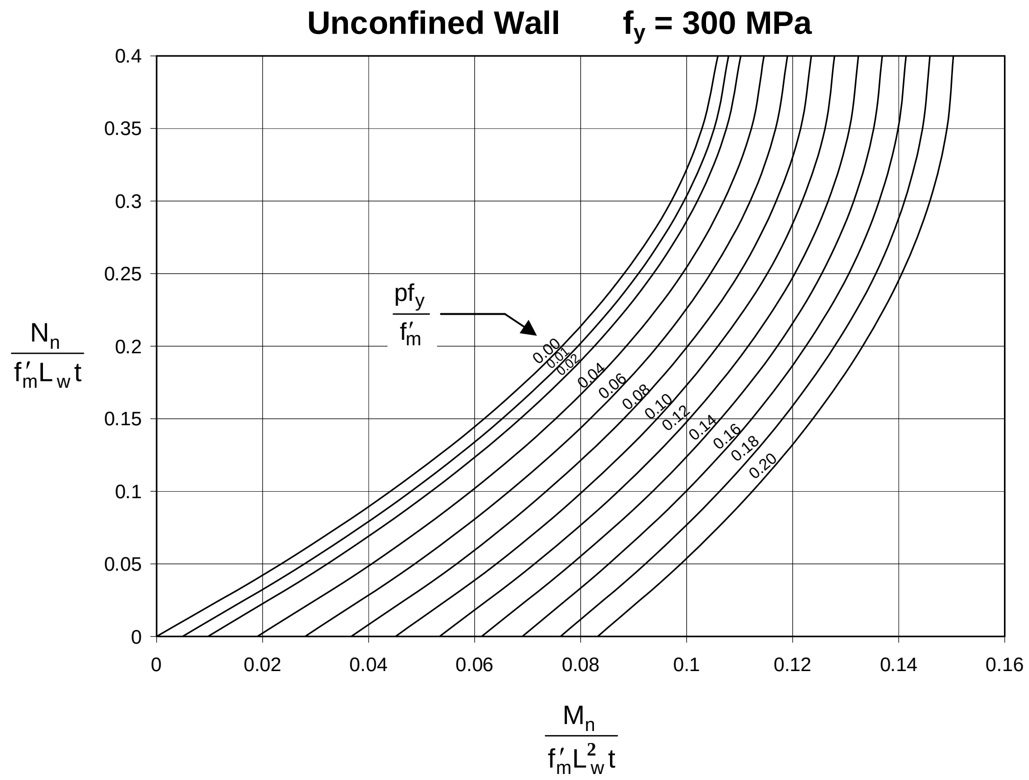

Figure 1: **Flexural Strength of Rectangular Masonry Walls with Uniformly Distributed Reinforcement, Unconfined Wall f_y = 300 MPa**

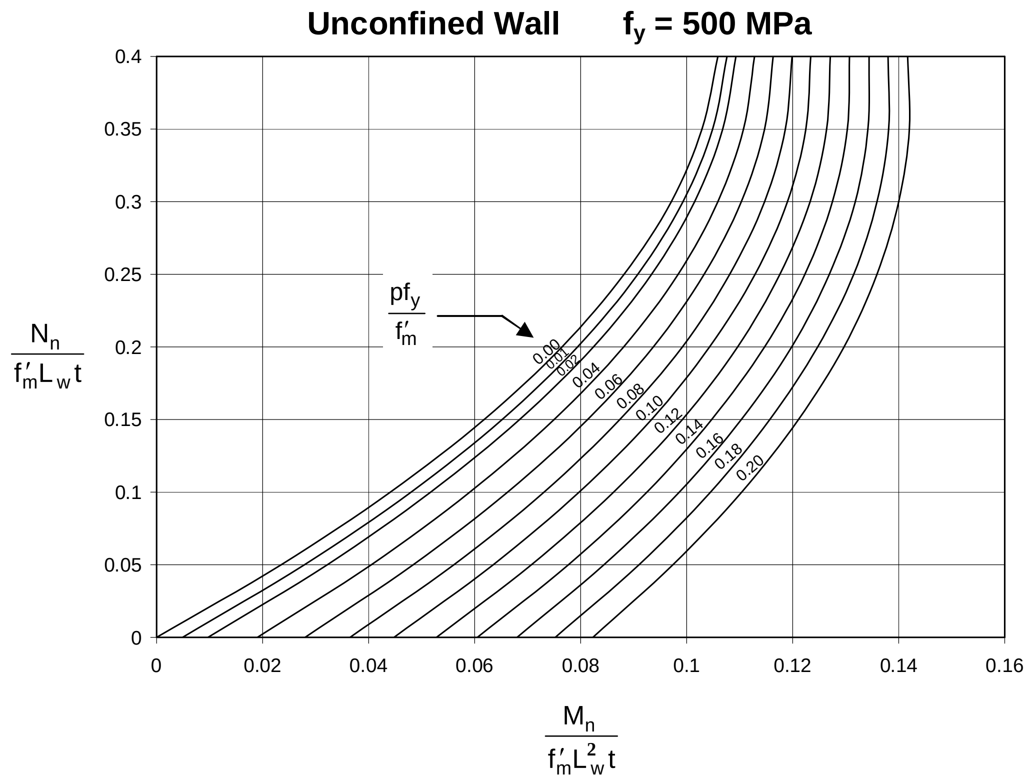

Figure 2: **Flexural Strength of Rectangular Masonry Walls with Uniformly Distributed Reinforcement, Unconfined Wall f_y = 500 MPa**

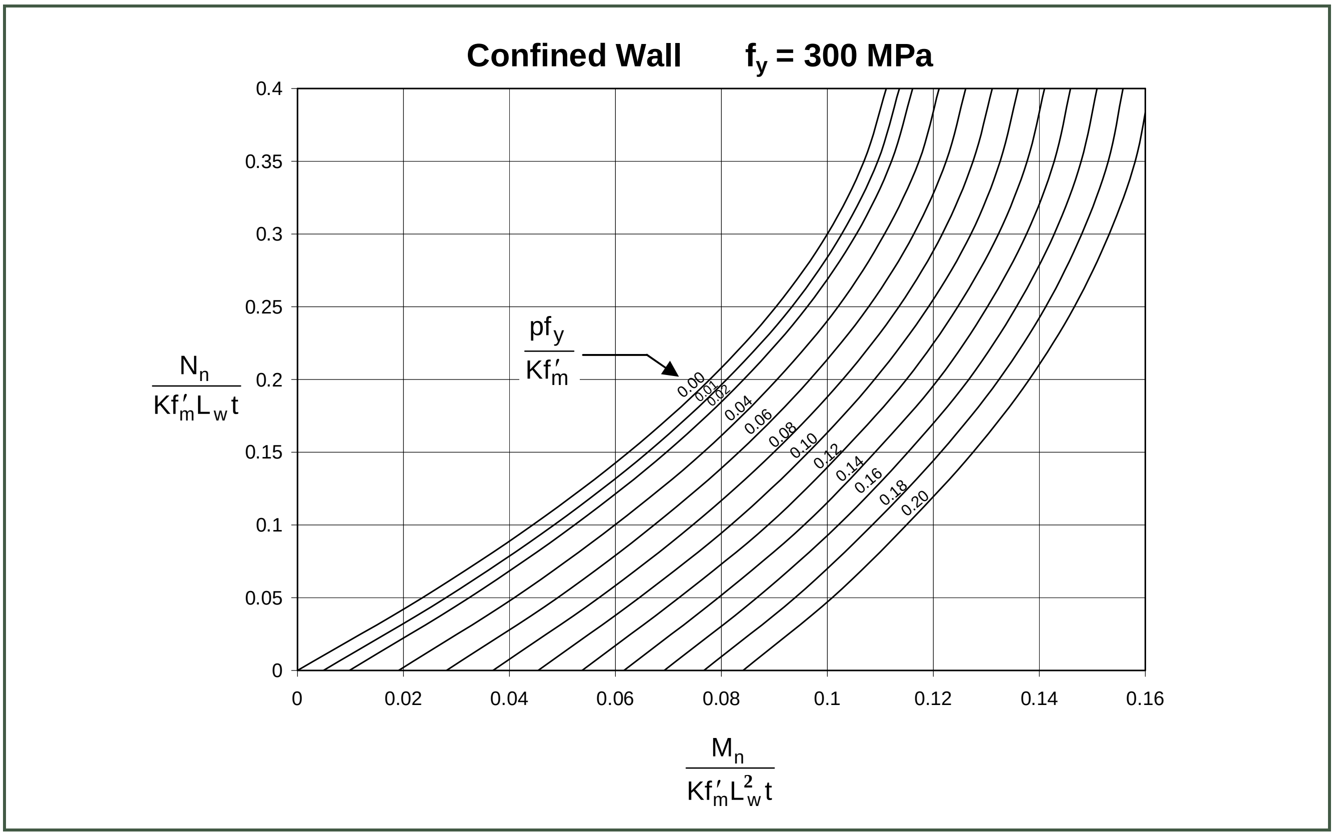

Figure 3: **Flexural Strength of Rectangular Masonry Walls with Uniformly Distributed Reinforcement, Confined Wall f_y = 300 MPa**

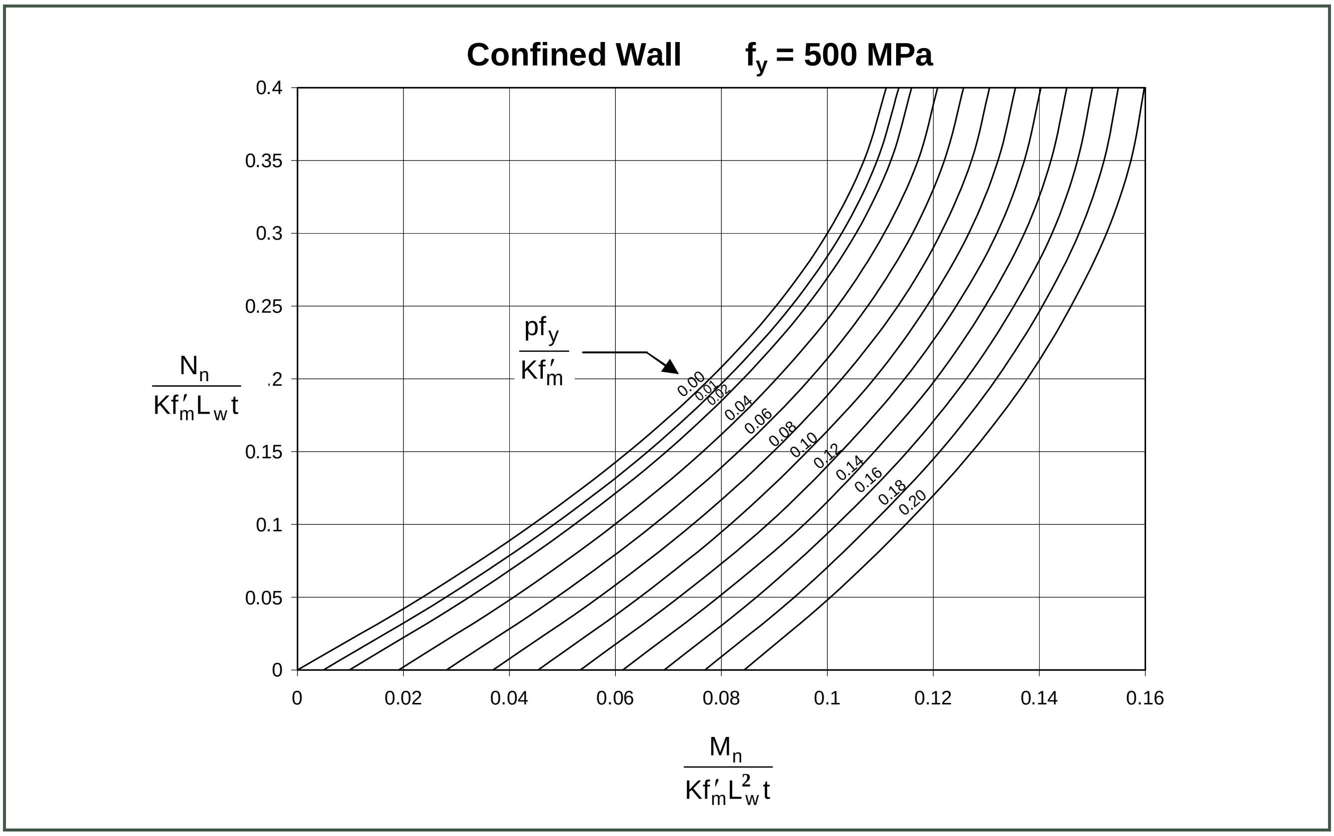

Figure 4: **Flexural Strength of Rectangular Masonry Walls with Uniformly Distributed Reinforcement, Confined Wall f_y = 500 MPa**

Table 6: **Neutral Axis Depth Ratio c/L_w (f_y = 300 MPa or 500 MPa): Unconfined Walls**

$\frac{pf_y}{f'_m}$	Axial Load Ratio $\frac{N_n}{f'_m L_w t}$								
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
0	0.0000	0.0692	0.1384	0.2076	0.2768	0.3460	0.4152	0.4844	0.5536
0.01	0.0135	0.0808	0.1481	0.2155	0.2828	0.3502	0.4175	0.4848	0.5522
0.02	0.0262	0.0918	0.1574	0.2230	0.2885	0.3541	0.4197	0.4852	0.5508
0.04	0.0498	0.1121	0.1745	0.2368	0.2991	0.3614	0.4237	0.4860	0.5483
0.06	0.0712	0.1306	0.1899	0.2493	0.3086	0.3680	0.4273	0.4866	0.5460
0.08	0.0907	0.1473	0.2040	0.2606	0.3173	0.3739	0.4306	0.4873	0.5439
0.1	0.1084	0.1626	0.2168	0.2710	0.3252	0.3794	0.4336	0.4878	0.5420
0.12	0.1247	0.1766	0.2286	0.2805	0.3325	0.3844	0.4364	0.4883	0.5403
0.14	0.1397	0.1895	0.2394	0.2893	0.3392	0.3890	0.4389	0.4888	0.5387
0.16	0.1535	0.2014	0.2494	0.2974	0.3453	0.3933	0.4412	0.4892	0.5372
0.18	0.1663	0.2125	0.2587	0.3048	0.3510	0.3972	0.4434	0.4896	0.5358
0.2	0.1782	0.2227	0.2673	0.3118	0.3563	0.4009	0.4454	0.4900	0.5345

Table 7: **Neutral Axis Depth Ratio c/L_w (f_y = 300 MPa or 500 MPa): Confined Walls**

$\frac{pf_y}{Kf'_m}$	Axial Load Ratio $\frac{N_n}{Kf'_m L_w t}$								
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
0	0.0000	0.0579	0.1157	0.1736	0.2315	0.2894	0.3472	0.4051	0.4630
0.01	0.0113	0.0679	0.1244	0.1810	0.2376	0.2941	0.3507	0.4072	0.4638
0.02	0.0221	0.0774	0.1327	0.1881	0.2434	0.2987	0.3540	0.4093	0.4646
0.04	0.0424	0.0953	0.1483	0.2013	0.2542	0.3072	0.3602	0.4131	0.4661
0.06	0.0610	0.1118	0.1626	0.2134	0.2642	0.3150	0.3659	0.4167	0.4675
0.08	0.0781	0.1270	0.1758	0.2246	0.2734	0.3223	0.3711	0.4199	0.4688
0.1	0.0940	0.1410	0.1880	0.2350	0.2820	0.3289	0.3759	0.4229	0.4699
0.12	0.1087	0.1540	0.1993	0.2446	0.2899	0.3351	0.3804	0.4257	0.4710
0.14	0.1224	0.1661	0.2098	0.2535	0.2972	0.3409	0.3846	0.4283	0.4720
0.16	0.1351	0.1774	0.2196	0.2618	0.3041	0.3463	0.3885	0.4307	0.4730
0.18	0.1471	0.1879	0.2288	0.2696	0.3105	0.3513	0.3922	0.4330	0.4739
0.2	0.1582	0.1978	0.2373	0.2769	0.3165	0.3560	0.3956	0.4351	0.4747

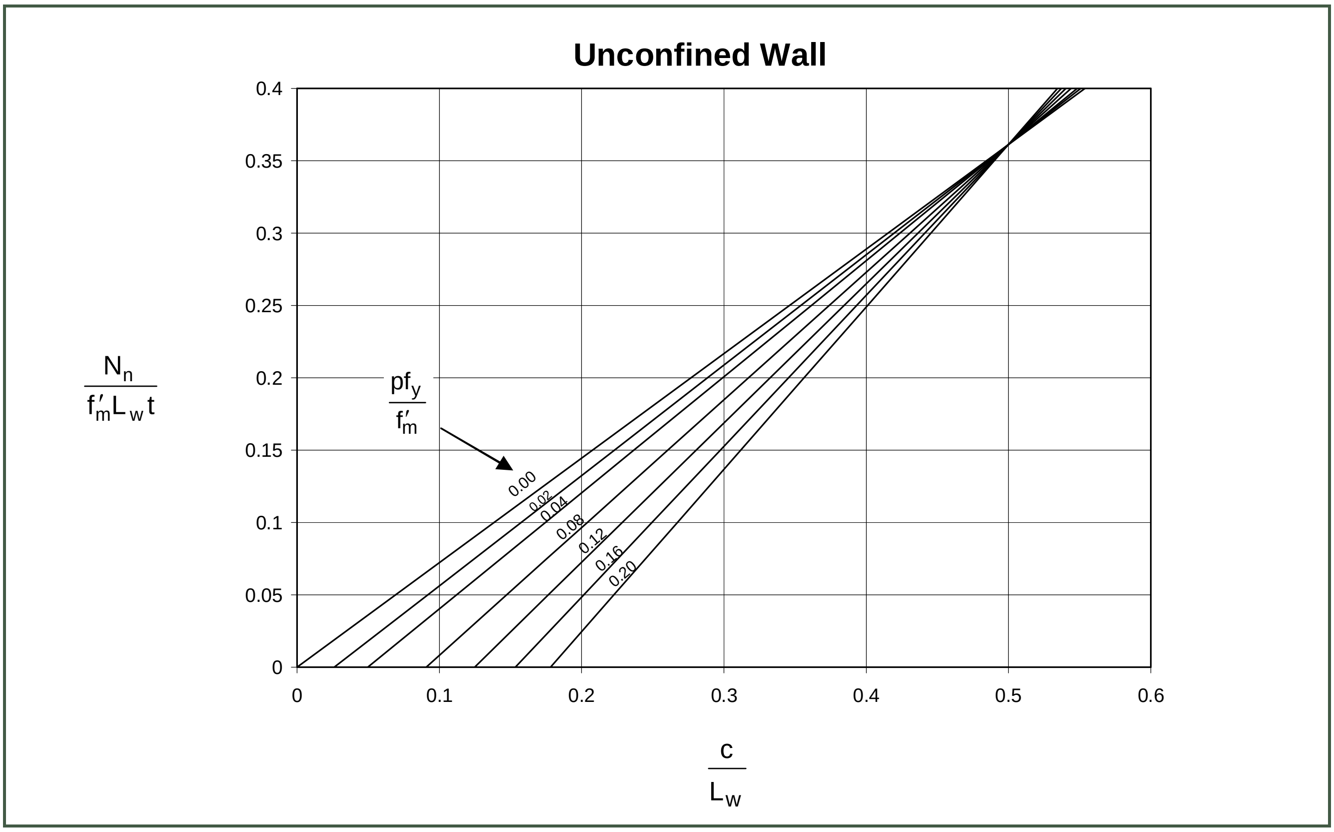

Figure 5: **Neutral Axis Depth of Unconfined Rectangular Masonry Walls with Uniformly Distributed Reinforcement, f_y = 300 MPa or 500 MPa**

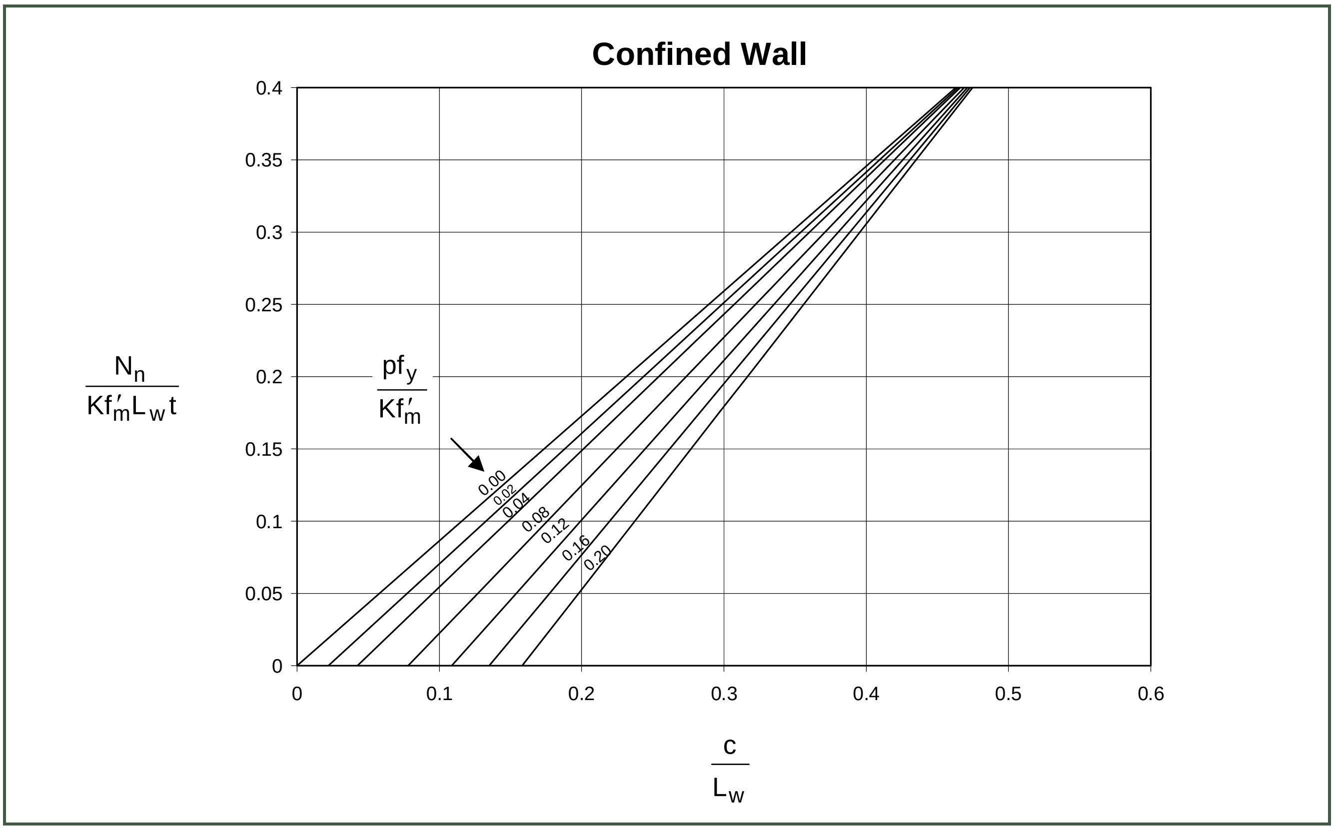

Figure 6: **Neutral Axis Depth of Confined Rectangular Masonry Walls with Uniformly Distributed Reinforcement, f_y = 300 MPa or 500 MPa**

2.7.2 Curvature Ductility

To avoid failure of potential plastic hinge regions of unconfined masonry shear walls, the masonry standard limits the extreme fibre compression strain at the full design inelastic response displacement to the unconfined ultimate compression strain of ε_u = 0.003. The available ductility at this ultimate compression strain decreases with increasing depth of the compression zone, expressed as a fraction of the wall length. Section 7.4.6 of NZS 4230:2004 ensures that the available ductility will exceed the structural ductility factor, μ, for walls of aspect ratio less than 3. This section provides justification for the relationship limiting neutral axis depth.

The most common and desirable sources of inelastic structural deformations are rotations in potential plastic hinges. Therefore, it is useful to relate section rotations per unit length (i.e. curvature) to corresponding bending moments. As shown in Figure 7(a), the maximum curvature ductility is expressed as:

$$\varepsilon_\phi = \frac{\phi_m}{\phi_y} \qquad [1]$$

where ϕ_m is the maximum curvature expected to be attained or relied on and ϕ_y is the yield curvature.

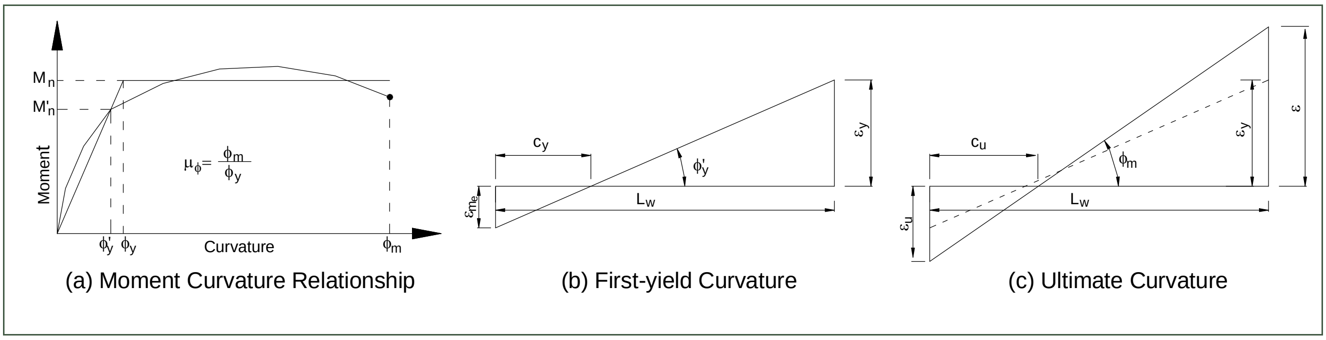

Figure 7: **Definition of curvature ductility**

Yield Curvature

For distributed flexural reinforcement, as would generally be the case for a masonry wall, the curvature associated with tension yielding of the most extreme reinforcing bar, ϕ'_y, will not reflect the effective yielding curvature of all tension reinforcement, identified as ϕ_y. Similarly, ϕ'_y may also result from nonlinear compression response at the extreme compression fibre.

$$\phi'_y = \frac{\varepsilon_y}{L_w - c_y} \quad \text{or} \quad \phi'_y = \frac{\varepsilon_y + \varepsilon_{me}}{L_w} \qquad [2]$$

where $\varepsilon_y = f_y / E_s$ and c_y is the corresponding neutral-axis depth. Extrapolating linearly to the nominal moment M_n, as shown in Figure 7(a), the yield curvature ϕ_y is given as:

$$\phi_y = \frac{M_n}{M'_n}\phi_y \qquad [3]$$

Maximum Curvature

The maximum attainable curvature of a section is normally controlled by the maximum compression strain ε_u at the extreme fibre. With reference to Figure 7(c), this curvature can be expressed as:

$$\phi_m = \frac{\varepsilon_u}{c_u} \qquad [4]$$

Displacement and Curvature Ductility

The displacement ductility for a cantilever concrete masonry wall can be expressed as:

$$\varepsilon_{\varphi} = \frac{\varphi}{\varphi_y} \quad \text{or} \quad \varepsilon_{\varphi} = \frac{\varphi_y + \varphi_p}{\varphi_y} \qquad [5]$$

consequently;

$$\varepsilon_{\varphi} = 1 + \frac{\varphi_p}{\varphi_y}$$

Yield Displacement

The yield displacement for a cantilever wall of height h_w may be estimated as:

$$\varphi_y = \phi_y h_w^2 / 3 \qquad [6]$$

Plastic Displacement

The plastic rotation occurring in the equivalent plastic hinge length L_p is given by:

$$\theta_p = \phi_p L_p = (\phi_m - \phi_y) L_p \qquad [7]$$

Assuming the plastic rotation to be concentrated at mid-height of the plastic hinge, the plastic displacement at the top of the cantilever wall is:

$$\varphi_p = \text{ɗ}_p (h_w - 0.5L_p) = (\phi_m - \phi_y) L_p (h_w - 0.5L_p) \qquad [8]$$

Substituting Eqns. 6 and 8 into Eqn. 5 gives:

$$\varepsilon_{\varphi} = 1 + \frac{(\phi_m - \phi_y) L_p (h_w - 0.5L_p)}{\phi_y h_w^2 / 3}$$

$$= 1 + 3(\varepsilon_{\phi} - 1) \frac{L_p}{h_w} \left(1 - L \frac{L_p}{2h_w}\right) \qquad [9]$$

Rearranging Eqn. 9:

$$\varepsilon_{\phi} = 1 + \frac{\varepsilon_{\varphi} - 1}{3(L_p/h_w)(1 - L_p/2h_w)} \qquad [10]$$

Paulay and Priestley (1992) indicated that typical values of the plastic hinge length is $0.3 < L_p/L_w < 0.8$. For simplicity, the plastic hinge length L_p may be taken as half the wall length L_w, and Eqn. 10 may be simplified to:

$$\varepsilon_{\phi} = 1 + \frac{\varepsilon_{\varphi} - 1}{\frac{3}{2}(L_w/h_w)(1 - L_w/4h_w)} \quad \text{or} \quad \varepsilon_{\phi} = 1 + \frac{\varepsilon_{\varphi} - 1}{\frac{3}{2A_r}\left(1 - \frac{1}{4A_r}\right)} \qquad [11]$$

where A_r is the wall aspect ratio h_w/L_w.

Reduced Ductility

The flexural overstrength factor $\phi_{o,w}$ is used to measure the extent of any over- or undersign:

$$\phi_{o,w} = \frac{\text{flexural overstrength}}{\text{moment resulting from loading Standard forces}} = \frac{M_{o,w}}{M_E^*} \qquad [12]$$

Whenever $\phi_{o,w}$ exceeds ∂_o/ϕ, the wall possesses reserve strength as higher resistance will be offered by the structure than anticipated when design forces were established. The overstrength factors λ_o are taken as 1.25 and 1.40 for grade 300 and 500 reinforcement respectively, while the strength reduction factor ϕ shall be taken as 0.85. It is expected that a corresponding reduction in ductility demand in the design earthquake will result. Consequently, design criteria primarily affected by ductility capacity may be met for the reduced ductility demand ($\varepsilon_{\varphi r}$) rather than the anticipated ductility (ε_{φ}). Therefore:

$$\varepsilon_{\varphi r} = \frac{\partial_o/\phi}{\phi_{o,w}}\varepsilon_{\varphi} \qquad [13]$$

2.7.3 Ductility Capacity of Cantilevered Concrete Masonry Walls

Section 7.4.6.1 of NZS 4230:2004 provides a simplified but conservative method to ensure that adequate ductility can be developed in masonry walls. The Standard allows the rational analysis developed by Priestley[3, 4] as an alternative to determine the available ductility of cantilevered concrete masonry walls.

Figure 8 includes dimensionless design charts for the ductility capacity, μ_3 of unconfined concrete masonry walls whose aspect ratio is $A_r = h_w/L_w = 3$. For walls of other aspect ratio, A_r, the ductility capacity can be found from the μ_3 value using Eqn. 14:

$$\varepsilon_{A_r} = 1 + \frac{3.3(\varepsilon_3 - 1)\left(1 - \frac{0.25}{A_r}\right)}{A_r} \qquad [14]$$

When the ductility capacity found from Figure 8 and Eqn. 14 is less than that required, redesign is necessary to increase ductility. The most convenient and effective way to increase ductility is to use a higher design value of f'_m for Type A masonry. This change will reduce the axial load ratio N_n/f'_mA_g (where $N_n = N^*/\phi$) and the adjusted reinforcement ratio $p^* = p12/f'_m$ proportionally. From Figure 8, the ductility will therefore increase.

Where the required increase in f'_m cannot be provided, a second alternative is to confine the masonry within critical regions of the wall. The substantial increase in ductility capacity resulting from confinement is presented in Figure 9. A third practical solution is to increase the thickness of the wall.

In Figures 8 and 9, the reinforcement ratio is expressed in the dimensionless form p*, where:

for unconfined walls: $$p^* = \frac{12p}{f'_m}$$

for confined walls: $$p^* = \frac{14.42p}{Kf'_m}$$

and $$K = 1 + p_s\frac{f_{yh}}{f'_m}$$

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3 Priestley, M. J. N. (1981). "Ductility of Unconfined Masonry Shear Walls", Bulletin NZNSEE, Vol. 14, No. 1, pp. 3-11.

4 Priestley, M. J. N. (1982). "Ductility of Confined Masonry Shear Walls", Bulletin NZNSEE, Vol. 5, No. 1, pp. 22-26.

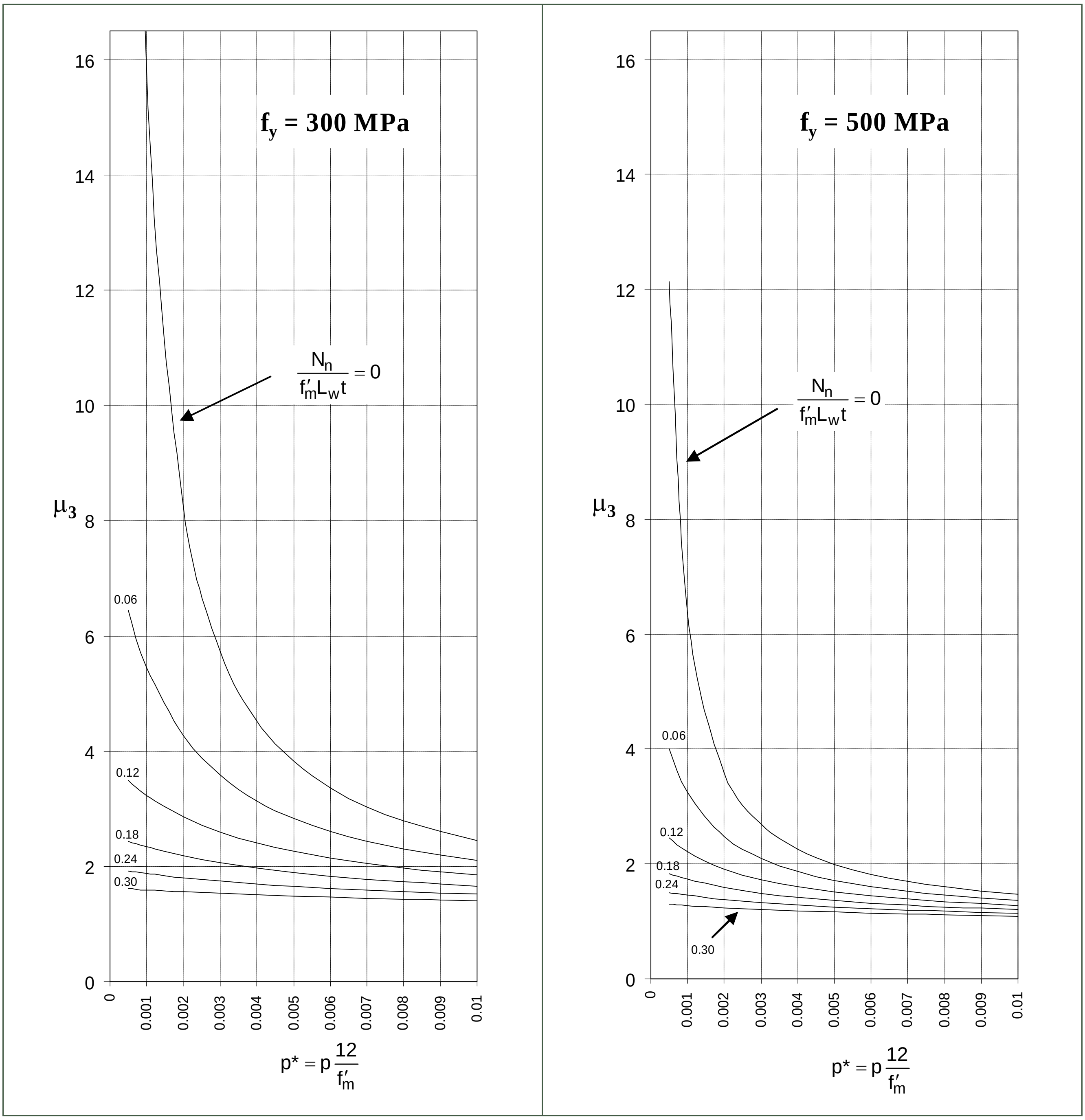


Figure 8: **Ductility of Unconfined Concrete Masonry Walls for Aspect Ratio A_r = 3**

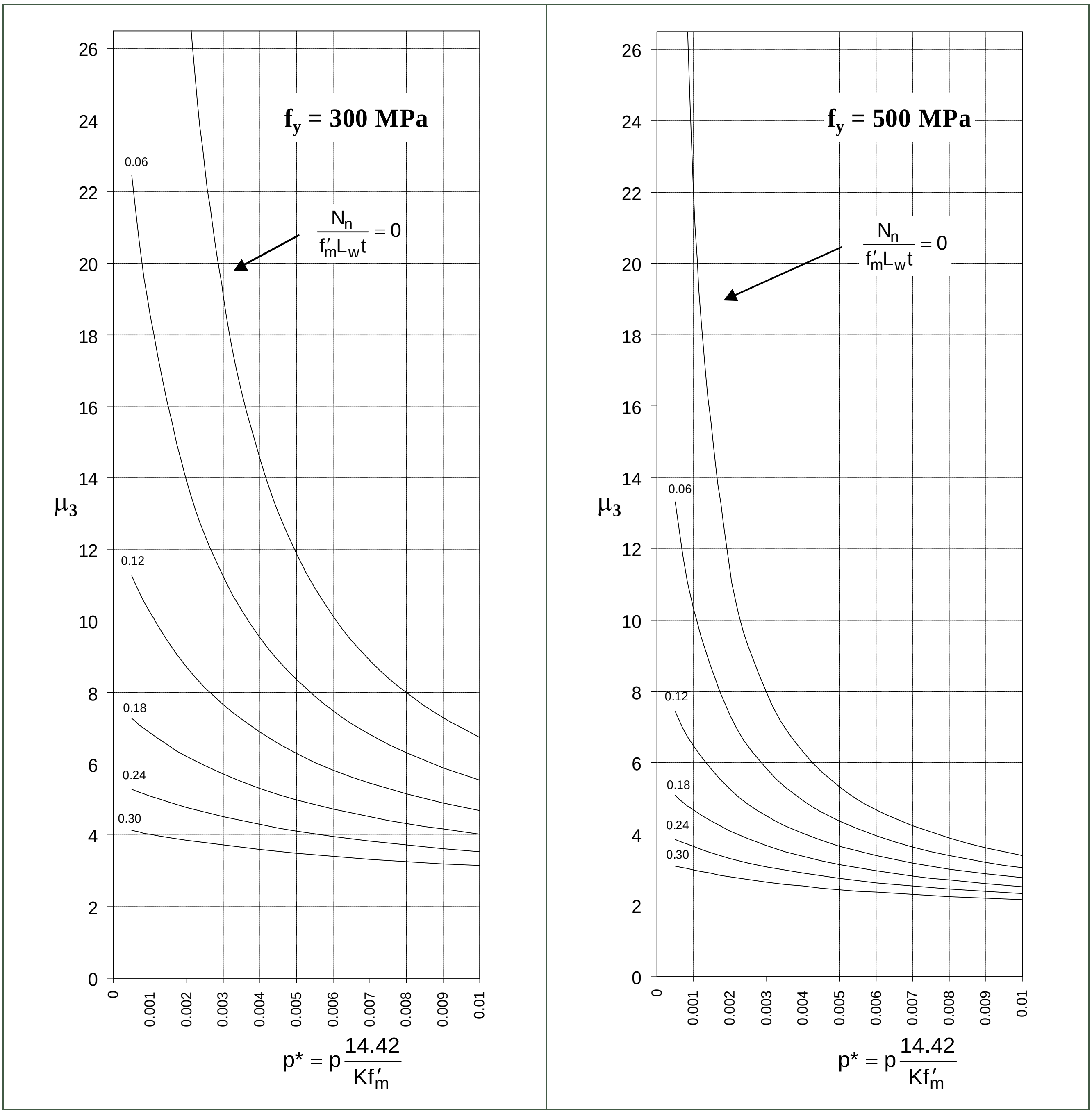


Figure 9: **Ductility of Confined Concrete Masonry Walls for Aspect Ratio A_r = 3**

2.7.4 Walls With Openings

Section 7.4.8.1 requires that for ductile cantilever walls with irregular openings, appropriate analyses such as based on strut-and-tie models shall be used to establish rational paths for the internal forces. Significant guidance on the procedure for conducting such an analysis is contained within NZS 3101, and an example is presented here in section 3.8.

2.8 Masonry In-plane Shear Strength

At the time NZS 4230:1990 was released, it was recognised that the shear strength provisions it contained were excessively conservative. However, the absence at that time of experimental data related to the shear strength of masonry walls when subjected to seismic forces prevented the preparation of more accurate criteria.

The shear resistance of reinforced concrete masonry components is the result of complex mechanisms, such as tension of shear reinforcement, dowel action of longitudinal reinforcement, as well as aggregate interlocking between the parts of the masonry components separated by diagonal cracks and the transmission of forces by diagonal struts forming parallel to shear cracks.

More recent experimental studies conducted in New Zealand and abroad have successfully shown the shear strength of reinforced masonry walls to be significantly in excess of that allowed by NZS 4230:1990. Consequently, new shear strength provisions are provided in section 10.3.2 of NZS 4230:2004. As outlined in clause 10.3.2.2 (Eqn. 10-5), masonry shear strength shall be evaluated as the sum of contributions from individual components, namely masonry (v_m), shear reinforcement (v_s) and applied axial compression load (v_p).

Masonry Component v_m

It has been successfully demonstrated through experimental studies that masonry shear strength, v_m increases with f'_m. However, the increase is not linear in all ranges of f'_m, but the rate becomes gradually lower as f'_m increases. Consequently, it is acceptable that v_m increases approximately in proportion to $\sqrt{f'_m}$. Eqn. 10-6 of NZS 4230:2004 is a shear expression developed by Voon and Ingham[5] for concrete masonry walls, taking into account the beneficial influence of the dowel action of tension longitudinal reinforcement and the detrimental influence of wall aspect ratio. These conditions are represented by the C_1 and C_2 terms included in Eqn. 10-6 of NZS 4230:2004. The v_{bm} specified in table 10.1 was established for a concrete masonry wall that has the worst case aspect ratio of $h_e/L_w \ge 1.0$ and reinforced longitudinally using grade 300 reinforcing steel with the minimum specified p_w of 0.07% (7.3.4.3).

For masonry walls that have aspect ratios of $0.25 \le h_e/L_w \le 1.0$ and/or p_w greater then 0.07%, the v_{bm} may be amplified by the C_1 and C_2 terms to give v_m. In order to guard against premature shear failure within the potential plastic hinge region of a component, the masonry standard assumes that little strength degradation occurs up to a component ductility ratio of 1.25, followed by a gradual decrease to higher ductility. This behaviour is represented by table 10.1 of NZS 4230:2004.

Axial Load Component v_p

Unlike NZS 4230:1990, the shear strength provided by axial load is evaluated independently of v_m in NZS 4230:2004. Section 10.3.2.7 of NZS 4230:2004 outlined the formulation, which considers the axial compression force to enhance the shear strength by arch action forming an inclined strut. Limitations of $v_p \le 0.1f'_m$ and $N^* \le 0.1f'_mA_g$ are included to prevent excessive dependence on v_p in a relatively squat masonry component and to avoid the possibility of brittle shear failure of a masonry component. In addition, the use of N^* when calculating v_p is to ensure a more conservative design than would arise using N_n.

Shear Reinforcement Component v_s

The shear strength contributed by the shear reinforcement is evaluated using the method incorporated in NZS 3101, but is modified for the design of masonry walls to add conservatism based on the perception that bar anchorage effects result in reduced efficiency of shear reinforcement in masonry walls, when compared with the use of enclosed stirrups in beams and columns.

As the shear strength provisions of NZS 4230:2004 originated from experimental data of masonry walls and because the new shear strength provisions generated significantly reduce shear reinforcement requirements, sections 8.3.11 and 9.3.6, and Eqn. 10-9 of NZS 4230:2004, must be considered to establish the quantity and detailing of minimum shear reinforcement required in beams and columns.

2.9 Design of Slender Wall

Slender concrete masonry walls are often designed as free standing vertical cantilevers, in applications such as boundary walls and fire walls, and also as simply supported elements with low stress demands such as exterior walls of single storey factory buildings. In such circumstances these walls are typically subjected to low levels

[5] Voon, K. C., and Ingham, J. M. (2007). "Design Expression for the In-plane Shear Strength of Reinforced Concrete Masonry", ASCE Journal of Structural Engineering, Vol. 133, No. 5, May, pp. 706-713.

of axial and shear stress, and NZS 4230:1990 permitted relaxation of the criteria associated with maximum wall slenderness in such situations.

At the time of release of NZS 4230:2004 there was considerable debate within the New Zealand structural design fraternity regarding both an appropriate rational procedure for determining suitable wall slenderness criteria, and appropriate prescribed limits for maximum wall slenderness (alternatively expressed as a minimum wall thickness for a prescribed wall height). This debate was directed primarily at the design of slender precast reinforced concrete walls, but it was deemed appropriate that any adopted criteria for reinforced concrete walls be applied in a suitably adjusted manner to reinforced concrete masonry walls.

Recognising that at the time of release of NZS 4230:2004 there was considerable ñengineering judgementò associated with the design of slender walls, the position taken by the committee tasked with authoring NZS 4230:2004 was to permit a minimum wall thickness of $0.05L_n$, where L_n is the smaller of the clear vertical height between effective line of horizontal support or the clear horizontal length between line of vertical support.

For free standing walls, an effective height of twice that of the actual cantilever height should be adopted. This $0.05L_n$ minimum wall thickness criteria, without permitting relaxation to $0.03L_n$ in special low-stress situations, is more stringent than the criteria provided previously in NZS 4230:1990, more stringent than that permitted in the US document TMS 402-11/ACI 530-11/ASCE 5-11, and more stringent than the criteria in NZS 3101:2006. Consequently, designers may elect to use ñengineering judgementò to design outside the scope of NZS 4230:2004, at their discretion. The appropriate criteria from these other documents is reported in Table 8 below.

Table 8: **Wall slenderness limits in other design standards**

Standard	Limits
NZS 4230:1990	Minimum wall thickness of $0.03L_n$ if: (a) Part of single storey structure, and (b) Elastic design for all load combinations, and (c) Shear stress less than $0.5v_n$
TMS 402-11/ACI 530-11/ASCE 5-11	Minimum wall thickness of $0.0333L_n$ if: (a) Factored axial compression stress less than $0.05f'_m$ (see section 3.3.5.3)
NZS 3101:2006	Minimum wall thickness of $0.0333L_n$ if: (a) $N^* > 0.2 f'_c A_g$ (section 11.3.7) Otherwise, more slender walls permitted (see NZS 3101:Part 1:2006, section 11.3 for further details)

3.0 Design Examples

3.1 Determine f'_m From Strengths of Grout and Masonry Units

Calculate the characteristic masonry compressive strength, f'_m, given that the mean strengths of concrete masonry unit and grout are 17.5 MPa and 22.0 MPa, with standard deviations of 3.05 MPa and 2.75 MPa respectively.

For typical concrete masonry, the ratio of the net concrete block area to the gross area of masonry unit is to be taken as 0.45, i.e. Ŭ = 0.45.

SOLUTION

The characteristic masonry compressive strength (5 percentile value) f'_m can be calculated from the strengths of the grout and the masonry unit using the equations presented in Appendix B of NZS 4230:2004.

Finding the mean masonry compressive strength, f_m

From Eqn. B-1 of NZS 4230:2004:

$$f_m = 0.59\,\breve{U} f_{cb} + 0.90(1-\breve{U})\,f_g$$

$$= 0.59 \times 0.45 \times 17.5 + 0.90 \times (1-0.45) \times 22.0$$

$$= 15.54 \text{ MPa}$$

Finding the standard deviation of masonry strength, x_m

From Eqn. B-2 of NZS 4230:2004:

$$x_m = \sqrt{0.35\breve{U}^2 x_{cb}^2 + 0.81(1-\breve{U})^2\, x_g^2}$$

$$= \sqrt{0.35 \times 0.45^2 \times 3.05^2 + 0.81 \times (1-0.45)^2 \times 2.75^2}$$

$$= 1.59 \text{ MPa}$$

Finding the characteristic masonry compressive strength, f'_m

From Eqn. B-3 of NZS 4230: 2004:

$$f'_m = f_m - 1.65 x_m$$

$$= 15.54 - 1.65 \times 1.59$$

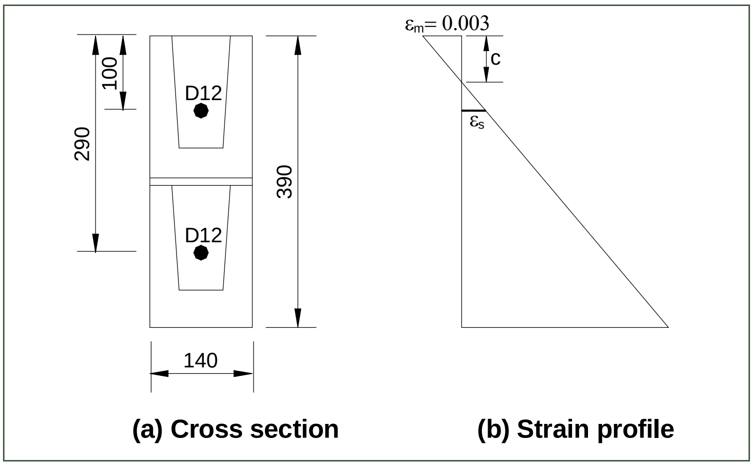
$$= 12.9 \text{ MPa}$$

Note that the values for mean and standard deviation of strength used here for masonry units and for grout correspond to the lowest characteristic values permitted by NZS 4210, with a resultant f'_m in excess of that specified in table 3.1 of NZS 4230:2004 for observation types B and A. Note also that these calculations have established a mean strength of approximately 15 MPa, supporting the use of E_m = 15 GPa as discussed here in section 2.3.2.

3.2 In-plane Flexure

3.2.1 3.2(a) Establishing Flexural Strength of Masonry Beam

Calculate the nominal flexural strength of the concrete masonry beam shown in Figure 10. Assume the beam is unconfined, f'_m = 12 MPa and f_y = 300 MPa.



(a) Cross section (b) Strain profile

Figure 10: **Concrete Masonry Beam**

SOLUTION

Assume that both D12 bars yield in tension. Therefore tension force due to reinforcement is:

$$A_s = ´ \times 12^2/4 \qquad = 113.1 \text{ mm}^2$$

$$\Rightarrow \qquad \Sigma T_i = \Sigma A_{si} f_y = 2 \text{ x } 113.1 \text{ x } 300 = 67.85 \text{ kN}$$

Now consider Force Equilibrium:

$$C_m = \Sigma T_i$$

where $C_m = 0.85 f'_m ab$

$$\Rightarrow \qquad 0.85 f'_m ab = 67.85 \text{ kN}$$

$$a = \frac{67.85 \times 10^3}{0.85\, f'_m \times 140} = 47.5 \text{ mm}$$

$$c = \frac{47.5}{0.85} = 55.9 \text{ mm}$$

Check to see if the upper reinforcing bar indeed yields:

$$\frac{\varepsilon_s}{100 - c} = \frac{\varepsilon_m}{c}$$

$$\Rightarrow \qquad \varepsilon_s = \frac{0.003}{55.9} \times 44.1 = 0.00237 > 0.0015 \qquad \text{therefore bar yielded}$$

Now taking moment about the neutral axis:

$$M_n = C_m \times (c - a/2) + T_i \times (d_i - c)$$

$$M_n = 67.85 \text{ x } (55.9 - 47.5/2) + 33.9 \text{ x } (100 - 55.9) + 33.9 \text{ x } (290 - 55.9)$$

$$= 11.6 \text{ kNm}$$

Alternatively, use Table 2 to establish flexural strength of the masonry beam:

$$p = \frac{A_s}{A_n} = \frac{226.2}{140 \times 390} = 0.0041$$

$$p\frac{f_y}{f'_m} = 0.0041 \times \frac{300}{12}$$

$$= 0.103$$

and $\dfrac{N_n}{f'_m A_n} = 0$

$$\Rightarrow \qquad \text{From Table 2,} \quad \frac{M_n}{f'_m h_b^2 t} \approx 0.0451$$

$$\Rightarrow \quad M_n = 0.0451 \times 12 \times 390^2 \times \frac{140}{1 \times 10^6}$$

$$M_n = 11.5 \text{ kNm}$$

3.2.2 3.2(b) Establishing Flexural Strength of Masonry Wall

Calculate the nominal flexural strength of the 140 mm wide concrete masonry wall shown in Figure 11. Assume the wall is unconfined, f'_m = 12 MPa, f_y = 300 MPa and N* = 115 kN.

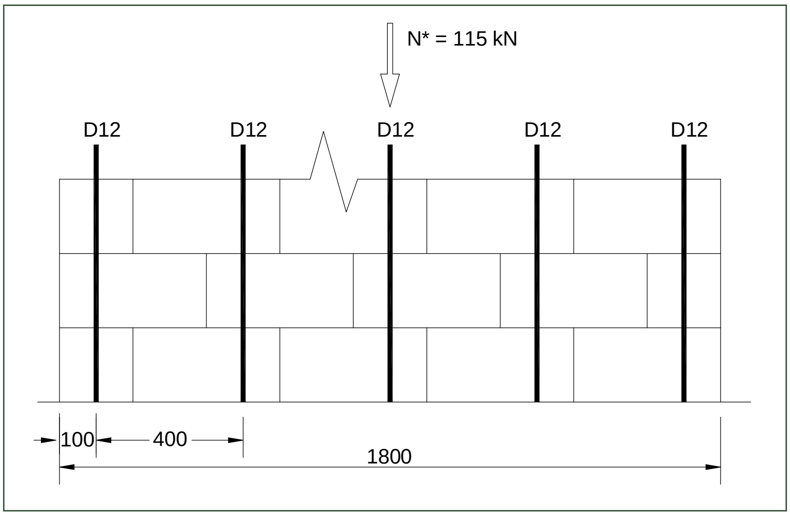
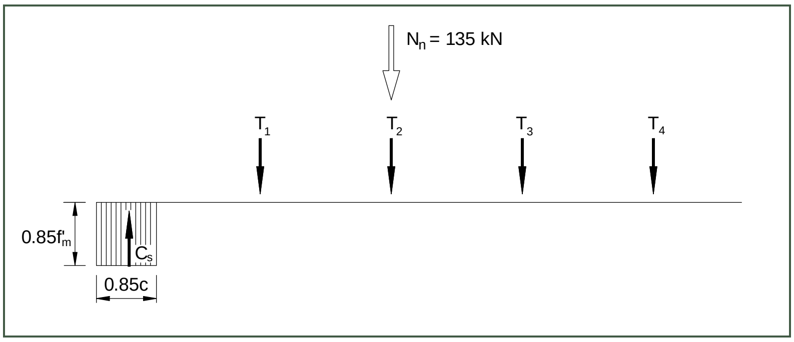


Figure 11: **Concrete Masonry Wall**

SOLUTION

Arial Load at Base

$$N_n = \frac{N^*}{\phi} = \frac{115}{0.85} = 135 \text{ kN}$$



Assume 4-D12 yield in tension and 1-D12 yields in compression:

$$\text{Area of 1-D12} = \pi \times \frac{12^2}{4} = 113.1 \text{ mm}^2$$

Therefore total tension force from longitudinal reinforcement:

$\Rightarrow$ T = 4 x 113.1 x 300 = 135.1 kN

and C_s = 113.1 x 300 = 33.9 kN

Now consider Force Equilibrium:

$C_m + C_s = T + N_n$

$C_m = T + N_n - C_s$ where $C_m = 0.85 f'_m ab$

$\Rightarrow \quad 0.85 f\hat{o}_m ab = 135.1 + 135$ ï 33.9

$\Rightarrow \quad 0.85 f\hat{o}_m ab = 236.8$ kN

$$a = \frac{236.8 \times 10^3}{0.85\, f'_m \times 140} = 165.8 \text{ mm}$$

$$c = \frac{165.8}{0.85} = 195.1\text{mm}$$

The reinforcing bar in compression is located closest to the neutral axis. Check to see that this bar does indeed yield:

$$\frac{\varepsilon_s}{c - 100} = \frac{\varepsilon_m}{c}$$

$$\Rightarrow \quad \varepsilon_s = \frac{0.003}{195.1} \times 95.1 = 0.00146 \text{ å } 0.0015 \quad \text{therefore OK}$$

Now taking moment about the neutral axis:

$$M_n = C_m \times \left(c - \frac{a}{2}\right) + T_i \times (d_i - c) + N_n \times \left(\frac{L_w}{2} - c\right)$$

$$M_n = 236.8 \text{ x } \left(195.1 - \frac{165.8}{2}\right) + 33.9 \text{ x } (195.1 - 100) + 33.9 \text{ x } (500 - 195.1) + 33.9 \text{ x } (900 - 195.1)$$

$$+ 33.9 \text{ x } (1300 - 195.1) + 33.9 \text{ x } (1700 - 195.1) + 135 \text{ x } \left(\frac{1800}{2} - 195.1\right)$$

$$= 247.7 \text{ kNm}$$

Alternatively, use Table 2 to establish flexural strength of the masonry wall:

$$p = \frac{A_s}{L_w t} = \frac{5 \times 113.1}{140 \text{ x } 1800} = 0.00224$$

$$p \frac{f_y}{f'_m} = 0.00224 \times \frac{300}{12} = 0.056$$

and $$\frac{N_n}{f'_m L_w t} = \frac{135 \times 10^3}{12 \times 1800 \times 140} = 0.045$$

$\Rightarrow$ From Table 2, $\dfrac{M_n}{f'_m L_w^2 t} \approx 0.04499$

$$M_n = 0.04499 \times 12 \times 1800^2 \times \frac{140}{1 \times 10^6}$$

$$= 245 \text{ kNm}$$

3.3 Out-of-Plane Flexure

A 190 mm thick fully grouted concrete masonry wall is subjected to N* = 21.3 kN/m and is required to resist an out-of-plane moment of M* = 17 kNm/m. Design the flexural reinforcement, using f'_m = 12 MPa and f_y = 300 MPa.

SOLUTION

Axial load: $$N_n = \frac{N^*}{\phi} = \frac{21.3}{0.85} \approx 25.0 \text{ kN/m}$$

Require $$M_n \geq \frac{M^*}{\phi}$$

$$\geq \frac{17}{0.85} = 20 \text{ kNm/m}$$

It is assumed that $M_n = M_p + M_s$, where M_p is moment capacity due to axial compression load N_n and M_s is moment capacity to be sustained by the flexural reinforcement.

As shown in Figure 12, moment due to N_n

$$a_1 = \frac{N_n}{0.85\, f'_m 1.0} = \frac{25 \times 10^3}{0.85 \times 12 \times 10^6}$$

$$= 2.45 \text{ mm}$$

Therefore $$M_p = N_n\left(\frac{t}{2} - \frac{a_1}{2}\right)$$

$$= 25 \times \left(\frac{190 - 2.45}{2}\right) = 2.34 \text{ kNm/m}$$

Now $$M_s = M_n - M_p$$

$$= 20 - 2.34$$

$$= 17.66 \text{ kNm/m}$$

Assuming $$a_2 \approx a_1 \frac{M_s}{M_p}$$

$$a_2 \approx 2.45 \times \frac{17.66}{2.34} \approx 18.5 \text{ mm}$$

$$M_s = A_s f_y \left(\frac{t}{2} - a_1 - \frac{a_2}{2}\right)$$

Therefore $$A_s \geq \frac{M_s}{f_y\left(\frac{t}{2} - a_1 - \frac{a_2}{2}\right)} = \frac{17.66 \text{ x } 10^3}{300 \text{ x } \left(95 - 2.45 - \frac{18.5}{2}\right) \text{ x } 10^3}$$

$$= 707 \text{ mm}^2\text{/m}$$

Try D20 reinforcing bars spaced at 400 mm c/c, A_s = 785 mm²/m

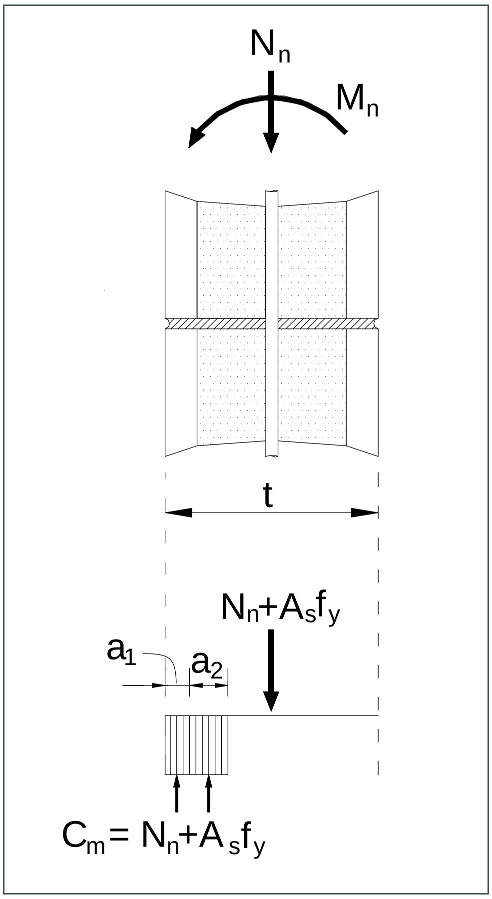


Figure 12: **Forces acting on wall**

Check

$$a = \frac{N_n + A_s f_y}{0.85\, f'_m\, 1.0} = \frac{25 \times 10^3 + 785 \times 300}{0.85 \times 12 \times 10^3} = 25.54 \text{ mm}$$

$$M_n = \left(N_n + A_s f_y\right) \times \left(\frac{t}{2} - \frac{a}{2}\right) = \left(25 \times 10^3 + 785 \times 300\right) \times \left(\frac{190 - 25.54}{2}\right) = 21.4 \text{ kNm/m} > \frac{M^*}{\phi}$$

3.4 Design of Shear Reinforcement

The single storey cantilevered concrete masonry wall of Figure 13 is to resist a shear force while responding elastically to the design earthquake. For a wall width of 140 mm, f'_m = 12 MPa and N* = 50 kN, design the required amount of shear reinforcement.

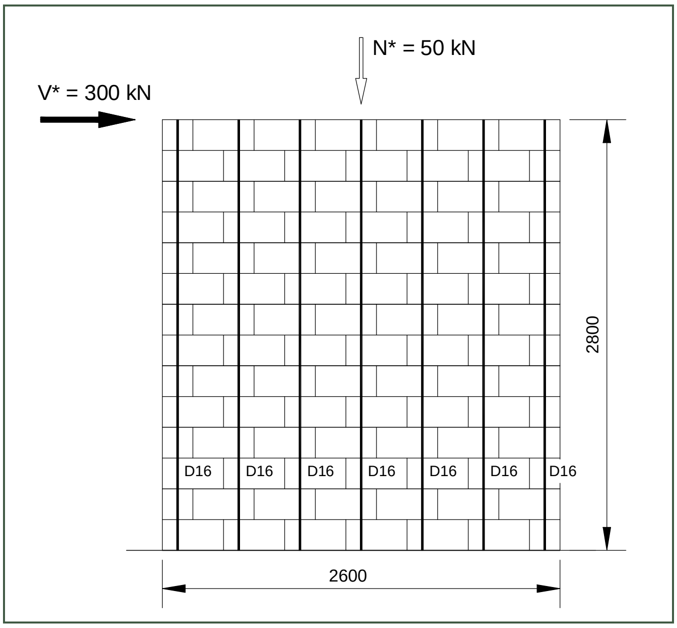


Figure 13: **Forces acting on masonry wall**

SOLUTION

N* = 50 kN

Therefore $N_n = \frac{N^*}{\phi} = \frac{50}{0.85} = 58.8$ kN

V* = 300 kN

Require $\phi V_n \geq V^*$

Therefore $V_n \geq \frac{V^*}{\phi}$

$$\geq \frac{300}{0.75}$$

$$\geq 400 \text{ kN}$$

Check maximum shear stress

$$V_n = \frac{V_n}{b_w d} \quad \text{note that } d = 0.8L_w \text{ for walls}$$

$$= \frac{400 \times 10^3}{140 \times 0.8 \times 2600}$$

$$= 1.37 \text{ MPa} < v_g$$

$$v_g = 1.50 \text{ MPa for } f'_m = 12 \text{ MPa}$$

Now $v_n = v_m + v_p + v_s$

Shear stress carried by v_m

$$v_m = (C_1 + C_2)v_{bm}$$

where $C_1 = 33p_w \dfrac{f_y}{300}$

and $p_w = \dfrac{7\text{bars} \times \text{D16}}{b_w d}$

$$= \frac{7 \times 201}{140 \times 0.8 \times 2600}$$

$$= 0.0048$$

$$\Rightarrow \quad C_1 = 33 \times 0.0048 \times \frac{300}{300}$$

$$= 0.16$$

$$C_2 = 1.0 \text{ since } h_e/L_w > 1.0$$

Hence, $v_m = (0.16 + 1.0)v_{bm}$ where $v_{bm} = 0.70$ MPa for $\mu = 1$ and $f'_m = 12$ MPa

$$\Rightarrow \quad v_m = 1.16 \times 0.70 = 0.81 \text{ MPa}$$

Shear stress carried by v_p

$$v_p = 0.9\frac{N^*}{b_w d}\tan Ŭ$$

where $N^* = 50$ kN

As illustrated in Figure 10.2 of NZS 4230:2004, it is necessary to calculate the compression depth **a** in order to establish tanŬ. The following illustrates the procedure of establishing compression depth **a** using Table 6:

$$p = \frac{7\text{bars} \times \text{D16}}{b_w \times L_w}$$

$$= \frac{7 \times 201}{140 \times 2600}$$

$$= 0.00387$$

$$p\frac{f_y}{f'_m} = 0.00387 \times \frac{300}{12} = 0.0967$$

and $$\frac{N_n}{f'_m L_w t} = \frac{58.8 \times 10^3}{12 \times 2600 \times 140} = 0.0135$$

From Table 6

$$\frac{c}{L_w} = 0.12$$

Therefore $c = 0.12 \times 2600$

$= 312$ mm

$\Rightarrow$ $a = \beta c$ (for unconfined concrete masonry, ß = 0.85)

$= 0.85 \times 312$

$= 265.2$ mm

Therefore $$\tan Ŭ = \frac{\frac{L_w}{2} - \frac{a}{2}}{h} = \frac{\frac{2600}{2} - \frac{265.2}{2}}{2800}$$

$= 0.417$

Hence, $$v_p = 0.9 \frac{50 \times 10^3}{140 \times 0.8 \times 2600} \times 0.417$$

$= 0.064$ MPa

Shear stress to be carried by v_s

$v_s = v_n - v_m - v_p = 1.37$ ï 0.81 ï 0.064

$= 0.50$ MPa

and $$v_s = C_3 \frac{A_v f_y}{b_w s}$$ where $C_3 = 0.8$ for a masonry walls

$\Rightarrow$ $$0.50 = 0.8 \frac{A_v \times 300}{140 \times 200}$$ Try $f_y = 300$ MPa and reinforcement spacing = 200 mm

$\Rightarrow A_v = 58.3$ mm^2

Therefore, use **R10 @ 200 crs** = 78.5 mm^2 per 200 mm spacing.

It is essential that shear reinforcement be adequately anchored at both ends, to be fully effective on either side of any potentially inclined crack. This generally required a hook or bend at the end of the reinforcement. Although hooking the bar round the end vertical reinforcement in walls is the best solution for anchorage, it may induce excessive congestion at end flues and result in incomplete grouting of the flue. Consequently bending the shear reinforcement up or down into the flue is acceptable, particularly for walls of small width.

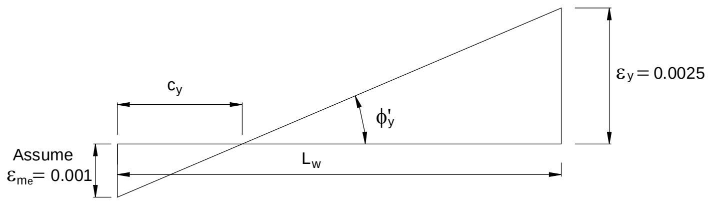
3.5 Concrete Masonry Wall Ductility Considerations

3.5.1 Neutral Axis of Limited Ductile Masonry Wall

Find the maximum allowable neutral axis depth for a limited ductile cantilever wall with aspect ratio of 3. The wall is reinforced with grade 500 reinforcement.

SOLUTION

$$\varepsilon_y = \frac{500}{200 \times 10^3} = 0.0025$$



For the purpose of an approximation that will generally overestimate the yield curvature, it may be assumed that ε_{me} = 0.001. This value would necessitate a rather large quantity of uniformly distributed vertical reinforcement in a rectangular wall, in excess of 1.5%. With this estimate the extrapolated yield curvature can be evaluated using Eqn. 2.

Using Eqn. 2: $$\phi'_y = \frac{0.0025 + 0.001}{L_w} = \frac{0.0035}{L_w}$$

Using Eqn. 3: $$\phi_y = \frac{M_n}{M'_n}\phi'_y \Rightarrow \phi_y \approx \frac{4}{3}\phi'_y = \frac{4}{3} x \frac{0.0035}{L_w}$$

Using Eqn. 11: $\varepsilon_\phi = 3.18$ for $\mu = 2$ and $h_e/L_w = 3$

Consequently;

$$\phi_m = \frac{\varepsilon_u}{c_{max}} = \mu_\phi \phi_y$$

$$= \frac{0.003}{c_{max}} = 3.18 x \frac{4}{3} x \frac{0.0035}{L_w}$$

$$\Rightarrow c_{max} = 0.202 L_w \qquad [15]$$

3.5.2 Neutral Axis of Ductile Masonry Wall

Find the maximum allowable neutral axis depth for a ductile cantilever wall (Aspect ratio of 3) reinforced with grade 500 reinforcement.

SOLUTION

Repeating the above exercise using Eqn. 11 we obtain $\varepsilon_\phi = 7.54$ for $\mu = 4$ and $h_e/L_w = 3$

Consequently;

$$\phi_m = \frac{\mathring{U}_u}{c_{max}} = \varepsilon_\phi \phi_y$$

$$\phi_m = \frac{0.003}{c_{max}} = 7.54 \text{ x } \frac{4}{3} \text{ x } \frac{0.0035}{L_w}$$

$$\Rightarrow c_{max} = 0.085\ L_w$$

3.6 Ductile Cantilever Shear Wall

The 6 storey concrete masonry shear wall of Figure 14 is to be designed for the seismic lateral loads shown, which have been based on a ductility factor of $\mu = 4.0$. Design gravity loads of 150 kN, including self weight, act at each floor and at roof level, and the weight of the ground floor and footing are sufficient to provide stability at the foundation level under the overturning moments. Wall width should be 190 mm. Design flexural and shear reinforcement for the wall.

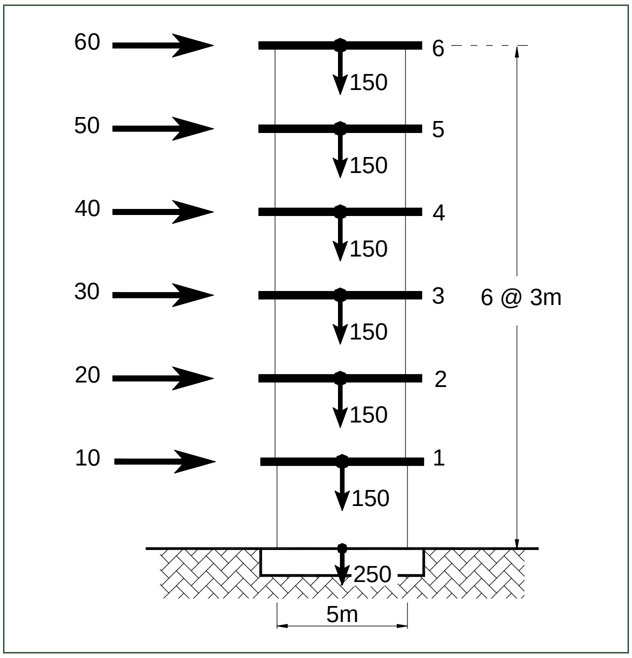


Figure 14: **Ductile Cantilever Shear Wall**

SOLUTION

Initially f'_m = 12 MPa will be assumed. From the lateral loads of Figure 14, the wall **base moment** is

M^* = 3 x (60 x 6 + 50 x 5 + 40 x 4 + 30 x 3 + 20 x 2 + 10)

= 2730 kNm

Require $\phi\, M_n\, \text{Ó} M^*$

Therefore $M_n\ \text{Ó} \frac{M^*}{\phi}$

$$M_n\ \text{Ó} \frac{2730}{0.85}$$

$$\text{Ó} 3211 \text{ kNm}$$

Axial Load at Base

$N^* = 6 \times 150 = 900$ kN

$$N_n = \frac{N^*}{\phi} = \frac{900}{0.85} = 1058.8 \text{ kN}$$

Check Dimensional Limitations

Assuming a 200 mm floor slab, the unsupported interstorey height = 2.8 m.

$$\frac{b_w}{L_n} = \frac{190}{2800} = 0.068 \qquad < 0.075$$

This is less than the general seismic requirement cited by the standard (clause 7.4.4.1). However, the axial load ratio is:

$$\frac{N_n}{f'_m L_w t} = \frac{1058.8 \times 10^3}{12 \times 5000 \times 190} = 0.093 \approx 0.1$$

And so from Table 6 we have

$c < 0.3 L_n$

Hence the less stringent demand of

$$\frac{b_w}{L_n} \geq 0.05$$

applies here (clause 7.3.3) and this is satisfied by the geometry of the wall.

Flexure and Shear Design

Dimensionless Design Parameters

$$\frac{M_n}{f'_m L_w^2 t} = \frac{3211.8 \times 10^6}{12 \times 5000^2 \times 190} = 0.0563$$

and $$\frac{N_n}{f'_m L_w t} = \frac{1058.8 \times 10^3}{12 \times 5000 \times 190} = 0.0929$$

From Figure 1 and assuming $f_y = 300$ MPa for flexural reinforcement

$$p\frac{f_y}{f'_m} = 0.04$$

Therefore $p = 0.0016$

Check Ductility Capacity

Check this using the ductility chart, Figure 8:

$$p\frac{12}{f'_m} = 0.0016 \quad \text{and} \quad \frac{N_n}{f'_m A_g} = 0.0929$$

Figure 6 gives $\mu_3 = 3.3$

Actual aspect ratio: $A_r = \frac{3 \times 6}{5} = 3.6$

Therefore from Eqn.14

$$\varepsilon_{3.6} = 1 + \frac{3.3 \times (3.3 - 1) \times \left(1 - \frac{0.25}{3.6}\right)}{3.6} = 3.0 < \mu = 4 \text{ assumed}$$

Thus ductility is inadequate and redesign is necessary

Redesign for f'_m =16 MPa. (Note that this will require verification of strength using the procedures reported in Appendix B of NZS 4230:2004).

Now New Dimensionless Design Parameters

$$\frac{N_n}{f'_m A_g} = \frac{1058.8 \times 10^3}{16 \times 5000 \times 190} = 0.0697$$

and $$\frac{M_n}{f'_m L_w^2 t} = \frac{3211.8 \times 10^6}{16 \times 5000^2 \times 190} = 0.0423$$

From Figure 1 and for f_y = 300 MPa for flexural reinforcement

$$p \frac{f_y}{f'_m} = 0.028$$

Therefore $$p = \frac{0.028 \times 16}{300} = 0.0015$$

Check Ductility Capacity

Using Figure 8, check the available ductility

$$p^* = p \frac{12}{f'_m} = 0.0015 \times \frac{12}{16} = 0.0011$$

$$\frac{N_n}{f'_m A_g} = 0.0697$$

From Figure 8, $\mu_3 \approx 4.5$

From Eqn. 14,

$$\varepsilon_{3.6} = 1 + \frac{3.3 \times (4.5 - 1) \times \left(1 - \frac{0.25}{3.6}\right)}{3.6} = 3.98 \approx 4.0$$

Hence ductility OK

Flexural Reinforcement

For p = 0.0015 reinforcement per 400 mm will be

$$A_s = 0.0015 \times 400 \times 190 = \frac{114 \text{ mm}^2}{400 \text{ mm}}$$

Therefore use **D12 @ 400 mm crs** (113 mm^2/400 mm).

Shear Design

To estimate the maximum shear force on the wall, the flexural overstrength at the base of the wall, M_o, needs to be calculated:

$M_o = 1.25 M_{n,provided}$ (for Grade 300 reinforcement)

$f'_m = 16$ MPa

$$\frac{N_n}{f'_m L_w t} = 0.070$$

$$p_{provided} = \frac{13 \text{ bars x } 113 \text{ mm}^2}{5000 \times 190} = 0.00155$$

and $$p\frac{f_y}{f'_m} = 0.00155 \times \frac{300}{16} = 0.029$$

From Table 2

$$\frac{M_n}{f'_m L_w^2 t} = 0.047$$

Therefore

$M_{n,provided} = 0.047 \text{ x } 16 \text{ x } 5000^2 \text{ x } 190 = 3580$ kNm

The overstrength value, $\phi_{o,w}$ is calculated as follows:

$$\phi_{o,w} = \frac{M_o}{M^*} = \frac{1.25\, M_{n,provided}}{M^*} = \frac{1.25 \times 3580}{2730} = 1.64$$

Dynamic Shear Magnification Factor

For up to 6 storeys:

$$\omega_v = 0.9 + \frac{n}{10}$$

$$= 0.9 + \frac{6}{10} = 1.5$$

Design of Reinforced Concrete Masonry Structures

Hence, the design shear force at the wall base is

$$V_n = \gamma_v \phi_{o,w} V^* = 1.5 \times 1.64 \times V^*$$

$$= 2.46V^*$$

$$= 2.46 \times 210$$

$$= 516.6 \text{ kN}$$

Check Maximum Shear Stress

$$v_n = \frac{V_n}{b_w d} = \frac{516.6 \times 10^3}{190 \times 0.8 \times 5000} = 0.68 \text{ MPa}$$

From Table 10.1 of NZS 4230:2004, the maximum allowable shear stress, v_g, for f'_m = 16 MPa is 1.8 MPa. Therefore OK.

Plastic Hinge Region

Within the plastic hinge region, $v_m = 0$. Therefore $v_p + v_s = 0.68$ MPa

and $$v_p = 0.9 \frac{N^*}{b_w d} \tan\alpha$$

As illustrated in Figure 10.2 of NZS 4230:2004, it is necessary to calculate the compression depth **a** in order to establish $\tan\alpha$.

To establish compression depth **a** using Table 6

$$p\frac{f_y}{f'_m} = 0.029 \quad \text{and} \quad \frac{N_n}{f'_m L_w t} = 0.0697$$

From Table 6

$$\frac{c}{L_w} = 0.126$$

Therefore $c = 0.126 \times 5000$

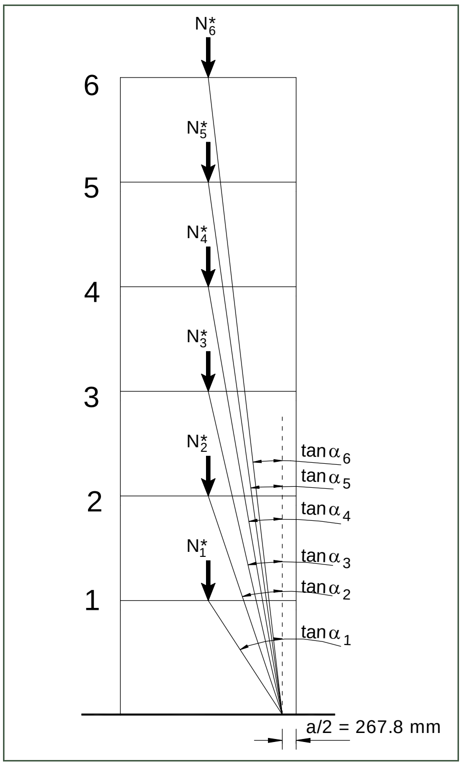
$= 630$ mm

$\Rightarrow$ $a = \beta c$ (for unconfined concrete masonry, $\beta = 0.85$)

$= 0.85 \times 630$

$= 535.5$ mm

Calculation of tanα



$$\tan\alpha_1 = \frac{2500 - 267.8}{3000} = 0.744$$

$$\tan\alpha_2 = \frac{2500 - 267.8}{6000} = 0.372$$

$$\tan\alpha_3 = \frac{2500 - 267.8}{9000} = 0.248$$

$$\tan\alpha_4 = \frac{2500 - 267.8}{12000} = 0.186$$

$$\tan\alpha_5 = \frac{2500 - 267.8}{15000} = 0.149$$

$$\tan\alpha_6 = \frac{2500 - 267.8}{18000} = 0.124$$

Figure 15: **Contribution of Axial Load**

Hence,

$$v_{p1} = 0.9\frac{150 \times 10^3}{190 \times 0.8 \times 5000} \times 0.744 = 0.112 \text{ MPa}$$

$$v_{p2} = 0.9\frac{150 \times 10^3}{190 \times 0.8 \times 5000} \times 0.372 = 0.066 \text{ MPa}$$

$$v_{p3} = 0.9\frac{150 \times 10^3}{190 \times 0.8 \times 5000} \times 0.248 = 0.044 \text{ MPa}$$

$$v_{p4} = 0.9\frac{150 \times 10^3}{190 \times 0.8 \times 5000} \times 0.186 = 0.033 \text{ MPa}$$

$$v_{p5} = 0.9\frac{150 \times 10^3}{190 \times 0.8 \times 5000} \times 0.149 = 0.026 \text{ MPa}$$

$$v_{p6} = 0.9\frac{150 \times 10^3}{190 \times 0.8 \times 5000} \times 0.124 = 0.022 \text{ MPa}$$

$$\Rightarrow \quad v_p = v_{p1} + v_{p2} + v_{p3} + v_{p4} + v_{p5} + v_{p6} = 0.30 \text{ MPa}$$

Therefore, the required shear reinforcement:

$v_s = v_n$ ï v_p

= 0.68 ï 0.30

= 0.38 MPa

$$v_s = C_3 \frac{A_v f_y}{b_w s}$$

where C_3 = 0.8 for a wall and the maximum spacing of transverse reinforcement = 200 mm since the wall height exceeds 3 storeys. Try f_y = 300 MPa.

$$0.38 = 0.8 \frac{A_v \times 300}{190 \times 200}$$

A_v = 60.2 mm^2/200 mm vertical spacing

Therefore use **R10 @ 200 crs** within plastic hinge region = 78.5 mm^2 per 200 mm spacing.

Outside Plastic Hinge Region

For example, immediately above level 2:

$$V_n = 1.5 \times 1.64 \times (60 + 50 + 40 + 30)$$
$$= 443 \text{ kN}$$

Therefore

$$v_n = \frac{443 \times 10^3}{b_w d} = 0.58 \text{ MPa}$$

From 10.3.2.6 of NZS 4230:2004

$$v_m = (C_1 + C_2) v_{bm}$$

where $C_1 = 33 p_w \dfrac{f_y}{300}$

$$= 33 \times \frac{13 \text{bars} \times 113}{b_w d} \frac{f_y}{300}$$

$$= 33 \times \frac{13 \times 113}{190 \times 0.8 \times 5000} \times \frac{300}{300}$$

$$= 0.064$$

and C_2 = 1.0 since $h_e/L_w > 1.0$

Therefore $v_m = (C_1 + C_2)v_{bm}$

$$= (0.064 + 1) \times 0.2\sqrt{16}$$

$$= 0.85 \text{ MPa} > v_n$$

Since $v_m > v_n$, only minimum shear reinforcement of 0.07% is required. Take s = 400 mm,

$$A_v = 0.07\% \times 400 \times 190 = 53.2 \text{ mm}^2$$

Therefore, use **R10 @ 400 crs** outside plastic hinge region.

3.7 Limited Ductile Wall with Openings

The seismic lateral loads for the 2 storey masonry wall of Figure 16 are based on the limited ductile approach, corresponding to μ = 2. Design gravity loads (both dead and live) including self weight are 20 kN/m at the roof, and 30 kN/m at levels 0 and 1. It is required to design the reinforce-ment for the wall, based on the limited ductility provisions of NZS 4230:2004, using f'_m = 16 MPa and f_y = 300 MPa. The wall thickness is 190 mm.

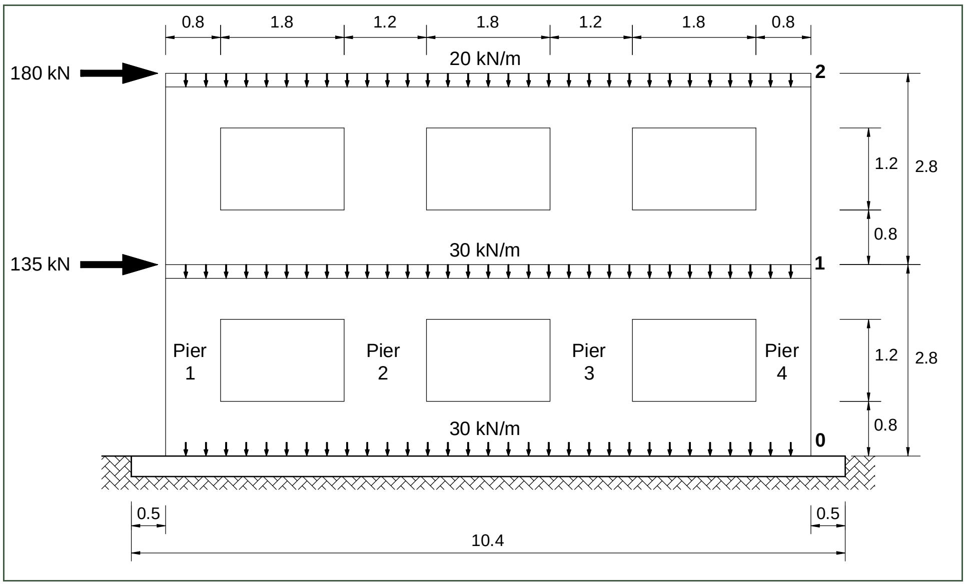


Figure 16: **Limited Ductile 2-Storey Masonry Wall with Openings.**

SOLUTION

As the structure is 2 storeys high, it may be designed for pier* hinging or spandrel* hinging as outlined in section 2.6.7.2(b) of NZS 3101:2006. Because of the relative proportions it is expected that pier hinging will initiate first, and this behaviour is assumed below. Consequently, the piers are identified as potential hinging areas. In accordance with section 3.7.3.3 of the standard, the spandrels are required to be designed for 50% higher moments than design level moments, with shear strength enhanced by 100% in spandrels and piers.

Axial Load

Assume each pier is loaded by the appropriate tributary area:

Axial load, 1st storey

Piers 1 and 4: N_{G+Qu} = (20 + 30) x (0.8 + 0.9) = 85 kN
Piers 2 and 3: N_{G+Qu} = 50 x (1.2 + 1.8) = 150 kN

Axial load, 2nd storey

Piers 1 and 4: N_{G+Qu} = 20 x (0.8 + 0.9) = 34 kN
Piers 2 and 3: N_{G+Qu} = 20 x (1.2 + 1.8) = 60 kN

Dimensional Limitations

Minimum thickness of piers:

b_w = 190 mm, L_n = 1200 mm

$$\frac{b_w}{L_n} = \frac{190}{1200} = 0.15$$

* Within this user guide, pier refers to the part of a wall or column between two openings, and spandrel refers to the deep beam above an opening.

This is more than the general seismic requirement of $b_w \geq 0.075L_n$ cited by the standard (7.4.4.1 of NZS 4230:2004).

Dimensional limitations of spandrels:

Spandrels at level 1 are more critical due to deeper beam depth. Therefore

b_w = 190 mm, h = 1600 mm and L_n = 1800 mm

$$\frac{L_n}{b_w} = \frac{1800}{190} = 9.5 \quad < 20$$

and $$\frac{L_n h}{b_w^2} = \frac{1800 \times 1600}{190^2} = 79.8 \quad < 80$$

The spandrels are within the dimensional limitations required by the standard (clause 8.4.2.3).

Determination of Seismic Lateral Forces in 1st Storey Piers

It is assumed that the spandrels are sufficiently stiff to force mid-height contraflexure points in the piers. The traditional approach of allocating lateral force to inelastically responding members in proportion to their assumed stiffness has been reported[6] to commonly lead to significant errors, regardless of whether gross stiffness or some fraction of gross stiffness is assumed. This is because walls of different length in the same direction will not have the same yield displacement. This can be illustrated by substituting Eqns. 2 and 3 into Eqn. 6 to give:

$$\Delta_y = \frac{M_n}{M'_n} \times \frac{\varepsilon_y + \varepsilon_m}{L_w} \times \frac{h_w^2}{3}$$

which indicates that the yield displacement is inversely proportional to wall length. This means that the basic presumption of the traditional approach, to allocate lateral load to walls in proportion to their stiffness as a means to obtain simultaneous yielding of the walls, and hence uniform ductility demand, is impossible to achieve. It was also shown by Paulay[7] that the yield curvature (ϕ_y) of a structural wall is insensitive to axial load ratio. As a consequence, it is possible to define ϕ_y as a function of wall length alone.

The moments and shears in the piers can be found from the method suggested by Paulay[7]. This design approach assigns lateral force between piers in proportion to the product of element area, $A_n = b_w L_w$, and element length, L_w, rather than the second moment of area of the section, as would result from a stiffness approach, i.e. the pier strength should be allocated in proportion to L_w^2 rather than L_w^3. Consequently the pier shear forces and moments are as summarised in Tables 9 and 10.

Table 9: **Pier Shear Forces**

Pier	Length, L_w (m)	L_w^2 (m^2)	$\frac{L_{wi}^2}{\Sigma L_{wi}^2}$	V_E (kN)	
				1st Storey	2nd Storey
1	0.8	0.64	0.154	48.5	27.7
2	1.2	1.44	0.346	109.0	62.3
3	1.2	1.44	0.346	109.0	62.3
4	0.8	0.64	0.154	48.5	27.7
	Σ	4.16	1.0	315	180

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6 Priestley, M. J. N., and Kowalsky, M. J. (1998) "Aspects of Drift and Ductility Capacity of Rectangular Cantilever Structural Walls", Bulletin of NZNSEE, Vol. 31, No. 2, pp. 73-85.

7 Paulay, T. (1997) "A Review of Code Provision for Torsional Seismic Effects in Buildings", Bulletin of NZNSEE, Vol. 30, No. 3, pp. 252-263.

Table 10: **Pier Shear Forces and Moments**

Parameter	Units	Pier 1	Pier 2	Pier 3	Pier 4	Σ
First Storey						
V^*_E	kN	48.5	109.0	109.0	48.5	315
M^*_E (1)	kNm	29.1	65.4	65.4	29.1	
M_{cl} (2)	kNm	67.9	152.6	152.6	67.9	
Second Storey						
V^*_E	kN	27.7	62.3	62.3	27.7	180
M^*_E (1)	kNm	16.6	37.4	37.4	16.6	
$M_{cl,\ top}$ (2)	kNm	27.7	62.3	62.3	27.7	
$M_{cl,\ bottom}$ (2)	kNm	38.8	87.2	87.2	38.8	

(1) Moments at critical pier i section
(2) Moments at spandrel centrelines, pier i

Note that in Table 10, the pier shear forces are used to establish the pier bending moments. For instance, the first storey bending moments of pier 1 are found from:

$$M_E^* = V_E^* \times \frac{h}{2} = 48.5 \times \frac{1.2}{2} = 29.1\,\text{kNm}$$

Spandrel moments and shears are found by extrapolating the pier moments to the pier/spandrel intersection points, then imposing moment equilibrium of all moments at a joint. At interior joints, the moments in the spandrels on either side of the joint are estimated, considering equilibrium requirements, by the assumption that the spandrel moment on one side of a joint centreline is equal to the ratio of the lengths of the adjacent span times the spandrel moment on the other side of the joint. For example, with regard to Figure 17b, at joint 2 the beam moment to the left of the centreline, M_{s21}, may be expressed as:

$$M_{s21} = \frac{\text{length of spandrel }(2-3)}{\text{length of spandrel }(1-2)} \times M_{s23} \qquad [16]$$

Hence

$$M_{s21} = \frac{\text{length of spandrel }(2-3)}{\text{length of spandrel }(1-2) + \text{length of spandrel }(2-3)} \times \sum \begin{pmatrix}\text{pier centreline} \\ \text{moments at joint 2}\end{pmatrix} \qquad [17]$$

More sophisticated analyses are probably inappropriate because of the deep members, large joints and influence of cracking and shear deformations. The resulting pier and spandrel moments and shears are plotted in Figure 17b. Axial forces in the piers are found from the resultant of beam shear (vertical equilibrium), and these are presented in Table 11.

Table 11: **Revised Total Axial Load**

Pier	$N^* = N_{G+Qu} + N_E$ (kN)	
	1st Storey	2nd Storey
1	85 - 103.8 = -18.8	34 - 21.4 = 12.6
2	156.5	61.3
3	143.5	58.7
4	188.8	55.4

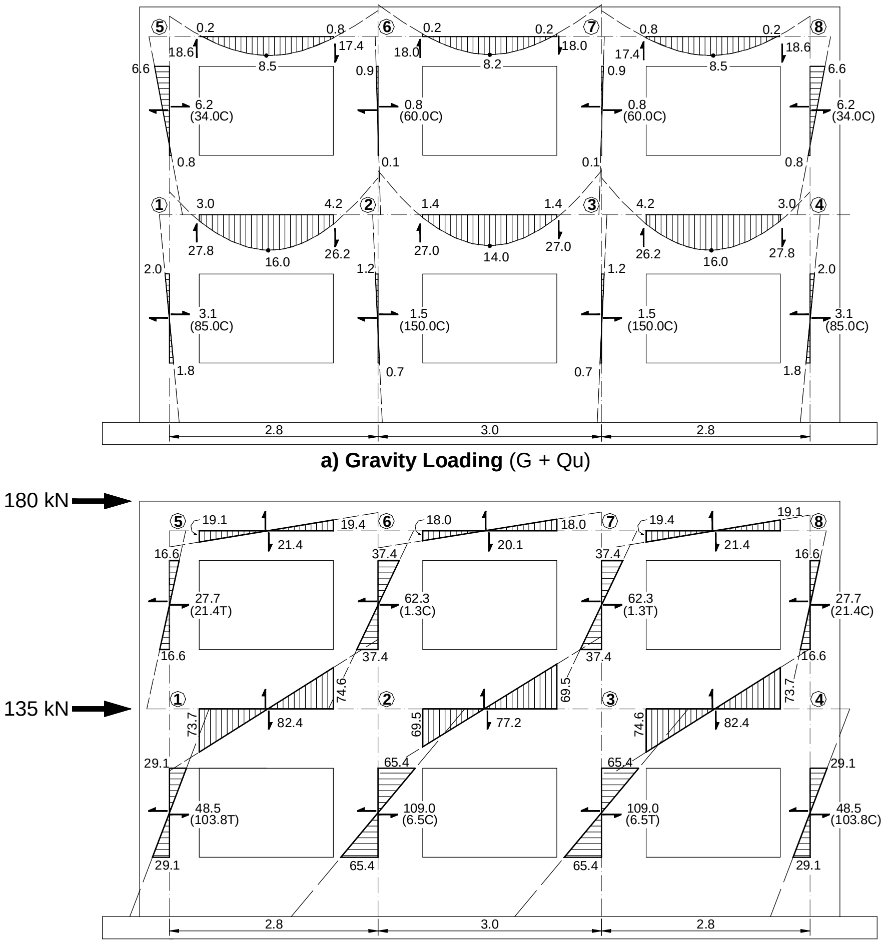

Figure 17: **Forces and Moments for the 2-Storey Masonry Wall** (Forces, Shears in kN, Moment in kNm, Axial Forces in parentheses)

Design of 1st Storey Piers

Flexural Design

Outer Piers

Outer piers are designed for the worst of Pier 1 and Pier 4 loading. Since the piers have been chosen as the ductile elements, the moments in Figure 17 are the design moments, i.e. $M^* = M^*_G + M^*_{Qu} + M^*_E$.

Pier 1

N* = -18.8 kN

$M^* = M^*_G + M^*_{Qu} + M^*_E$ = -2.0 + 29.1 = 27.1 kNm (Note that $M^*_G + M^*_{Qu}$ = -2.0 kNm)

Therefore

$$Nn = \frac{N^*}{\phi} = \frac{-18.8}{0.85}$$

= -22.1 kN

and $$M_n \geq \frac{M^*}{\phi}$$

$$M_n \geq \frac{27.1}{0.85}$$

$$\geq 31.9 \text{ kNm}$$

Dimensionless Design Parameters

$$\frac{N_n}{f'_m L_w t} = \frac{-22.1 \times 10^3}{16 \times 800 \times 190} = -0.0091$$

and $$\frac{M_n}{f'_m L_w^2 t} = \frac{31.9 \times 10^6}{16 \times 800^2 \times 190} = 0.0164$$

From Figure 1, $p\frac{f_y}{f'_m} = 0.037$

<u>Pier 4</u>

N* = 188.8 kN

$M^* = M_G^* + M_{Qu}^* + M_E^*$ = 2.0 + 29.1 = 31.1 kNm

Therefore

$$N_n = \frac{N^*}{\phi} = \frac{188.8}{0.85} = 222.1 \text{ kN}$$

and $$M_n \geq \frac{M^*}{\phi}$$

$$M_n \geq \frac{31.1}{0.85}$$

$$\geq 36.6 \text{ kNm}$$

Dimensionless Design Parameters

$$\frac{N_n}{f'_m L_w t} = \frac{222.1 \times 10^3}{16 \times 800 \times 190} = 0.091$$

and

$$\frac{M_n}{f'_m L_w^2 t} = \frac{36.6 \times 10^6}{16 \times 800^2 \times 190} = 0.0188$$

From Figure 1, $p\frac{f_y}{f'_m} < 0.00$

$\Rightarrow$ Pier 1 governs

Now $\quad p\dfrac{f_y}{f'_m} = 0.037$ for f_y = 300 MPa and f'_m = 16 MPa

$$\Rightarrow p = \frac{0.037 \times 16}{300} = 0.002$$

As the structure is designed as one of limited ductility, the requirements of clause 7.4.5.1 of NZS 4230:2004 apply for spacing and bar size. Consequently, it is required to adopt minimum bar size of D12 and minimum of 4 bars, i.e. 200 crs. With D12 at 200 crs, $p = \dfrac{\pi \times 12^2}{4 \times 200 \times 190}$ = 0.00297. This exceeds the p = 0.002 required. Refer to Figure 18 for details.

Inner Piers

Inner piers are designed for the worst loading conditions of Piers 2 and 3.

From Figure 17, it may be determined that Pier 3 governs design due to larger bending moment and lighter axial compressive load.

Pier 3:

N^* = 143.5 kN

$M^* = M^*_G + M^*_{Qu} + M^*_E = 1.2 + 65.4 = 66.6$ kNm

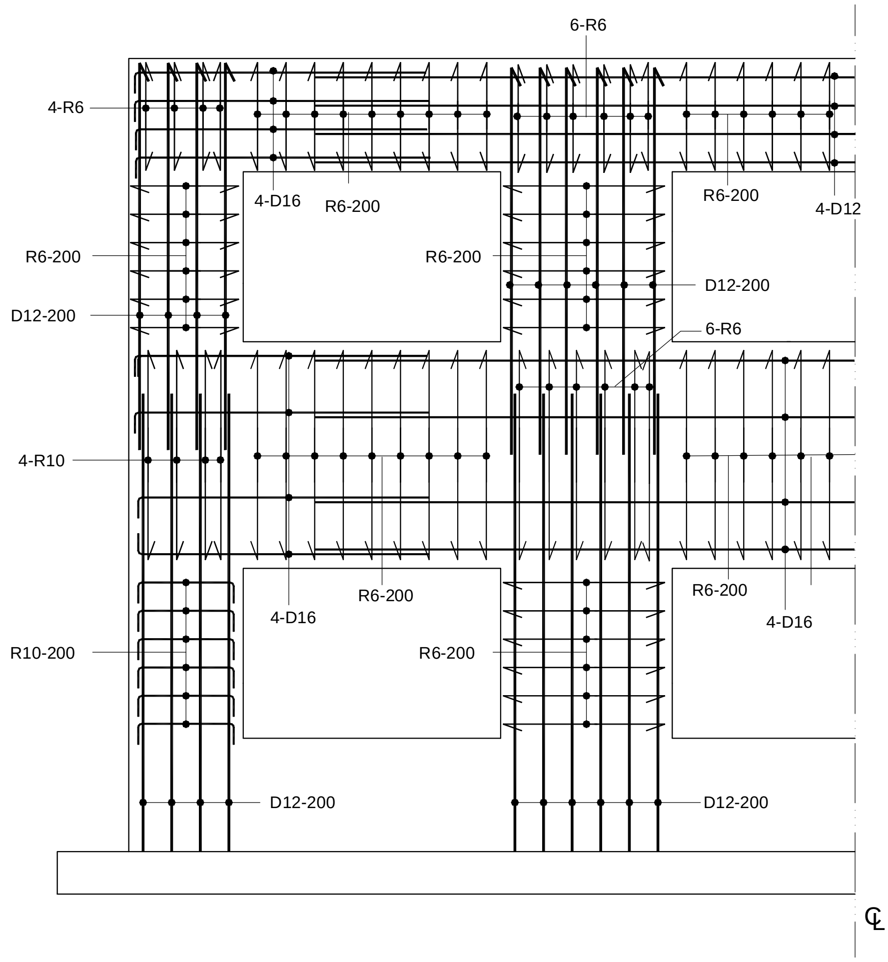

Figure 18: **Reinforcement for Design Example 3.7**

Therefore

$$N_n = \frac{N^*}{\phi} = \frac{143.5}{0.85}$$

$$= 168.8 \text{ Kn}$$

and $$M_n \geq \frac{M^*}{\phi}$$

$$M_n \geq \frac{66.6}{0.85}$$

$$\geq 78.2 \text{ kNm}$$

Dimensionless Design Parameters

$$\frac{N_n}{f'_m L_w t} = \frac{168.8 \times 10^3}{16 \times 1200 \times 190} = 0.046$$

and $$\frac{M_n}{f'_m L_w^2 t} = \frac{78.2 \times 10^6}{16 \times 1200^2 \times 190} = 0.0179$$

From Figure1, $p\dfrac{f_y}{f'_m} \approx 0.00$

Therefore use D12 @ 200 for the two inner piers to satisfy the requirements of clause 7.4.5.1.

Refer to Figure 18 for details.

Ductility Checks

Clause 7.4.6.1 of NZS 4230:2004 requires that for walls with contraflexure point between adjacent heights of lateral support:

$$c \leq 0.45 L_w^2 / L_n$$

where L_w is the wall length, and L_n is the unsupported height.

Note that calculations should be conducted using the amount of reinforcement required ($p_{required}$) rather than the amount of reinforcement actually provided, as the latter results in a higher moment capacity, and hence reduced ductility demand, for which a higher value of **c** could be tolerated.

Pier	$c_{max} = \dfrac{0.45L_w^2}{L_n}$	$\dfrac{N_n}{f'_m A_g}$	$p_{required} \dfrac{f_y}{f'_m}$	$c_{required}$ (from Table 6)	
1	240	-0.006	0.040	40	OK
2	540	0.050	0.000	83	OK
3	540	0.042	0.000	70	OK
4	240	0.082	0.000	91	OK
Units	mm	---	---	mm	

Shear Design, 1st Storey

From NZS 4230:2004: $\phi V_n \geq V_G^* + V_{Qu}^* + 2V_E^*$ where ϕ = 0.75

Outer Piers

Pier 1 governs due to the presence of axial tension force, $V^* = -3.1 + 2 \times 48.5 =$ 93.9 kN (where $V_G^* + V_{Qu}^* = -3.1\,\text{kN}$ and $V_E^* = 48.5$ kN)

$$\Rightarrow V_n = \frac{93.9}{0.75} = 125.2\text{ kN}$$

Now for Type A masonry, $v_g = 0.45\sqrt{f'_m}$ = $0.45 \times \sqrt{16}$ = 1.8 MPa

Check shear stress, b_w = 190 mm, d = 0.8 x 800 = 640 mm

$$v_n = \frac{V_n}{b_w d} = \frac{125.2 \times 10^3}{190 \times 640} = 1.03\text{ MPa} \qquad Ò v_g$$

From Section 10.3 of NZS 4230:2004:

$$v_n = v_m + v_p + v_s$$

Shear stress carried by $v_m = (C1 + C2)v_{bm}$

where $C_1 = 33 p_w \dfrac{f_y}{300}$

and $p_w = 0.00297$

$$\Rightarrow C_1 = 33 \times 0.00297 \times \frac{300}{300} = 0.098$$

and $C_2 = 0.42\left[4 - 1.75\left(\dfrac{h_e}{L_w}\right)\right]$

$$\Rightarrow C_2 = 0.42\left[4 - 1.75 \times 1200/(2 \times 800)\right]$$

$$\Rightarrow C_2 = 1.12$$

Hence,

$v_m = (0.098 + 1.12)v_{bm}$ where $v_{bm} = 0.15\sqrt{f'_m}$ for ε = 2.

$$\Rightarrow v_m = 0.73\text{ MPa}$$

***Shear stress carried by* $\mathbf{v_p = 0.9 \dfrac{N^*}{b_w d} \tan\alpha}$**

where N^* = -18.8 kN

$$\Rightarrow N_n = \frac{N^*}{\phi} = -22.1\text{ kN}$$

and $p = 0.00297$

$$\frac{N_n}{f'_m L_w t} = -0.0091$$

and $p\frac{f_y}{f'_m} = 0.0557$

From Table 6, $\frac{c}{L_w} \approx 0.068$

For Pier 1 with L_w = 800 mm,

$\Rightarrow c = 54.4$ mm

Therefore, a = 0.85 x c = 46.2 mm

Consequently, for pier in double bending $\tan\alpha = \frac{800-46.2}{1200} = 0.628$

$$\Rightarrow v_p = 0.9 \times \frac{-18.8\times10^3}{190\times0.8\times800} \times 0.628 = -0.087 \text{ MPa}$$

Shear stress to be carried by $v_s = v_n - v_m - v_p$

$v_s = v_n - v_m - v_p = 1.03 - 0.73 - (-0.087)$

$= 0.39$ MPa

and $v_s = C_3 \frac{A_v f_y}{b_w s}$ where C_3 = 0.8 for masonry walls

$\Rightarrow 0.39 = 0.8\frac{A_v \times 300}{190\times200}$ Try f_y = 300 MPa and reinforcement spacing = 200 mm

$\Rightarrow A_v = 61.8 \text{ mm}^2$

Therefore, use <u>R10 @ 200 crs</u> (78.5 mm^2)

Inner Piers

Clearly, Pier 3 governs due to lighter compression load, $V^* = 1.5 + 2\times109.0 = 219.5$ kN

$$V_n = \frac{219.5}{0.75} = 292.7 \text{ kN}$$

Check shear stress, b_w = 190 mm, d = 0.8 x 1200 = 960 mm

$$\Rightarrow v_n = \frac{V_n}{b_w d} = \frac{292.7\times10^3}{190\times960} = 1.60 \text{ MPa} \quad < v_g$$

Shear stress carried by $v_m = (C_1 + C_2)v_{bm}$

where $C_1 = 33p_w \dfrac{f_y}{300}$

and $p_w = 0.00297$

$\Rightarrow C_1 = 0.098$

and $C_2 = 0.42\left[4 - 1.75\left(\dfrac{h_e}{L_w}\right)\right]$

$\Rightarrow C_2 = 0.42\left[4 - 1.75 \times 1200/(2 \times 1200)\right]$

$\Rightarrow C_2 = 1.31$

Hence,

$v_m = (0.098 + 1.31) \times 0.15\sqrt{16}$ where $v_{bm} = 0.15\sqrt{f'_m}$ for $\mu = 2$.

$\Rightarrow v_m = 0.84$ MPa

Shear stress carried by $\mathbf{v_p = 0.9\dfrac{N^*}{b_w d}\tan\alpha}$

where $N^* = 143.5$ Kn

$\Rightarrow N_n = \dfrac{143.5}{0.85} = 168.8$ kN

and $p = 0.00297$

$\dfrac{N_n}{f'_m L_w t} = 0.046$

and $p\dfrac{f_y}{f'_m} = 0.0557$

From Table 6, $\dfrac{c}{L_w} = 0.122$

For Pier 3 with $L_w = 1200$, $c = 0.122 \times 1200 = 146.4$ mm

Therefore $a = 0.85 \times 146.4 = 124.4$ mm

Consequently $\tan\breve{U} = \dfrac{1200 - 124.4}{1200} = 0.90$ for pier in double bending

$\Rightarrow v_p = 0.9 \times \dfrac{143.5 \times 10^3}{190 \times 0.8 \times 1200} \times 0.90 = 0.64$ MPa

Shear stress to be carried by $v_s = v_n - v_m - v_p$

$v_s = v_n - v_m - v_p$ = 1.60 ï 0.84 ï 0.64

= 0.12 MPa

$$v_s = C_3 \frac{A_v f_y}{b_w s}$$ where C_3 = 0.8 for masonry walls

$$\Rightarrow 0.12 = 0.8 \frac{A_v \times 300}{190 \times 200}$$ Try f_y = 300 MPa and reinforcement spacing = 200 mm

$$\Rightarrow A_v = 19.0 \text{ mm}^2$$

However, this is less than the p_{min} = 0.07% required by clause 7.3.4.3 of the standard. Therefore, use R6 @ 200 crs (28.2 mm^2) to give p = 0.074%.

Design of 2nd Storey

The procedure is the same as for 1st storey and is not repeated here. Minimum requirements of D12 @ 200 again govern flexure, but shear reinforcement in the outer piers can be reduced to 0.07% of the gross cross-sectional area of the wall (minimum reinforcement area required by clause 7.3.4.3).

Flexural Design, Level 2 Spandrels

Section 3.7.3 of NZS 4230:2004 requires

$$\phi M_n \geq M_G^* + M_{Qu}^* + 1.5 M_E^*$$

Spandrels 1-2 and 3-4

Design for the maximum moments adjacent to Joint 3, $M_G^* + M_{Qu}^* = 4.2$ kNm and $M_E^* = 74.6$ kNm.

Therefore $M^* = 4.2 + 1.5 \times 74.6$ = 116.1 kNm

Note that beam depth = 1.6 m and N* = 0

$$M_n = \frac{116.1}{0.85} = 136.6 \text{ kNm}$$

Dimensionless Design Parameter

$$\frac{M_n}{f'_m L_w^2 t} = \frac{136.6 \times 10^6}{16 \times 1600^2 \times 190} = 0.0176$$

From Table 2,

$$p \frac{f_y}{f'_m} = 0.037$$

$$\Rightarrow p = \frac{0.037 \times 16}{300} = 0.00197$$

Therefore use D16 @ 400 crs (average p = 0.00265), i.e. cells 1, 3, 6 and 8 from top. See Figure 18 for details.

Spandrel 2-3

Design for the maximum moment of $M^* = 1.4 + 1.5 \times 69.5$ = 105.7 kNm, adjacent to Joint 2.

Therefore $M_n = \frac{105.7}{0.85}$ = 124.4 kNm

Dimensionless Design Parameter

$$\frac{M_n}{f'_m L_w^2 t} = \frac{124.4 \times 10^6}{16 \times 1600^2 \times 190} = 0.016$$

From Table 2,

$$p \frac{f_y}{f'_m} = 0.034$$

$$\Rightarrow p = \frac{0.034 \times 16}{300} = 0.0018$$

Therefore continue D16 @ 400 crs right through Spandrel 2-3.

Shear Design, Level 2 Spandrels

Design requirement $\phi V_n \geq V_G^* + V_{Qu}^* + 2V_E^*$, and ϕ = 0.75 for shear

Spandrels 1-2 and 3-4

$$V^* = 27.8 + 2 \times 82.4 = 192.6 \text{ kN (adjacent to Joint 4)}$$

$$V_n = \frac{192.6}{0.75} = 256.8 \text{ kN}$$

$$\Rightarrow v_n = \frac{V_n}{b_w d} = \frac{256.8 \times 10^3}{190 \times 0.8 \times 1600} = 1.06 \text{ MPa} \quad < v_g$$

Since beams are assumed not to be hinging (pier flexural demand, ϕM_n, was met, therefore flexural capacity of spandrels has an additional reserve strength of $1.5M_E^*$). Consequently, $v_{bm} = 0.2\sqrt{f'_m}$, see Table 10.1 of NZS 4230:2004.

$$v_m = (C_1 + C_2)v_{bm}$$

where $C_1 = 33p_w \frac{f_y}{300}$ note that p_w = 0.00265

$$\Rightarrow C_1 = 0.087$$

and C_2 = 1 for beams

$$\Rightarrow v_m = (0.087 + 1) \times 0.2\sqrt{16} = 0.87 \text{ MPa}$$

Therefore $v_s = v_n - v_m - v_p$

and $v_p = 0$

$\Rightarrow \quad v_s = 1.06 - 0.87 - 0$

$= 0.19$ MPa

$$v_s = C_3 \frac{A_v f_y}{b_w s} \qquad \text{note that } C_3 = 1.0 \text{ for beams}$$

Clause 10.3.2.10 requires spacing of shear reinforcement, placed perpendicular to the axis of component not to exceed 0.5d or 600 mm.

Therefore, maximum shear reinforcement spacing, $s_{max} = 600$ mm

$\Rightarrow$ Try s = 200 mm and $f_y = 300$ MPa

$$v_s = \frac{A_v \times 300}{190 \times 200}$$

$$\Rightarrow \quad 0.19 = \frac{A_v \times 300}{190 \times 200}$$

$\Rightarrow \quad A_v = 24.1\ \text{mm}^2$

Use R6 @ 200 crs (i.e. $A_v = 28\ \text{mm}^2$ per 200 mm). This is also the minimum area of reinforcement of 0.07% required by clause 7.3.4.3 of the standard.

Spandrels 2-3

$$V^* = 27.0 + 2 \times 77.2 = 181.4 \text{ kN}$$

$$V_n = \frac{181.4}{0.75} = 241.9 \text{ kN}$$

$$\Rightarrow \quad v_n = \frac{V_n}{b_w d} = \frac{241.9 \times 10^3}{190 \times 0.8 \times 1600} = 0.99 \text{ MPa} \quad < v_g$$

$$v_m = (C_1 + C_2) v_{bm}$$

where $C_1 = 33 p_w \frac{f_y}{300}$ note that $p_w = 0.00265$

$\Rightarrow C_1 = 0.087$

and $C_2 = 1.0$ for beams

$$\Rightarrow \quad v_m = (0.087 + 1) \times 0.2\sqrt{16} = 0.87 \text{ MPa}$$

Therefore $v_s = v_n - v_m - v_p$

$\Rightarrow \quad v_s = 0.99 - 0.87 - 0 \qquad$ (note that $v_p = 0$)

$v_s = 0.12$

$$v_s = \frac{A_v \times 300}{190 \times 200} \qquad \text{Try s = 200 mm and } f_y = 300 \text{ MPa}$$

$$\Rightarrow \qquad 0.12 = \frac{A_v \times 300}{190 \times 200}$$

$$\Rightarrow \qquad A_v = 15.2 \text{ mm}^2$$

Therefore use R6 @ 200 crs.

Design of Level 3 Spandrels

The design of level 3 spandrels is similar to above and is not included herein.

Beam-Column Joints

Check dimensional limitations

Minimum vertical dimension, h_b:

Interior joints (11.4.2.3a of NZS 4230:2004):

h_b = 1600 mm

d_{bc} = 12 mm

$$\text{Therefore} \quad \frac{h_b}{d_{bc}} = \frac{1600}{12} = 133 \quad > 70$$

Exterior joints (11.4.2.5):

h_b = 800 mm

d_{bc} = 12 mm

$$\text{Therefore} \quad \frac{h_b}{d_{bc}} = \frac{800}{12} = 67$$ This is about 4% shortfall of the requirement, therefore OK

Minimum horizontal dimension, h_c:

Interior joints (11.4.2.2b):

h_c = 1200 mm
d_{bb} = 16 mm

$$\text{Therefore} \quad \frac{h_c}{d_{bb}} = \frac{1200}{16} = 75 \quad > 60$$

Exterior joints (11.4.2.4):

$$\begin{aligned} \text{required} \quad h_c &= \text{cover} + L_{dh} + 10d_{bb} \\ &= 100 + 20d_b + 10d_{bb} \\ &= 100 + (20 \times 16) + (10 \times 16) \\ &= 580 \text{ mm} < h_c \text{ provided is 800 mm, therefore OK.} \end{aligned}$$

Joint Shear Design

The joints should be designed to the provisions of Section 11 of NZS 4230:2004. At level 2, the critical joints are 3 and 4. If there is doubt as to the critical joints then it is prudent to evaluate **all** joints.

An estimation of the joint shear force may be found by the appropriate slope of the moment gradient through the joint (Paulay and Priestley, 1992). Hence, the horizontal shear V_{jh} and vertical shear V_{jv} at a joint are approximated by:

$$V_{jh} \approx \frac{M_t + M_b - \frac{(V_{bL} + V_{bR})h'_c}{2}}{h'_b}$$

$$V_{jv} \approx \frac{M_L + M_R - \frac{(V_{col\,t} + V_{col\,b})h'_b}{2}}{h'_c}$$

where M_t, M_b, M_L and M_R are the moments at top, bottom, left and right of the joint. V_{bL} and V_{bR} are the shears applied to the left and right sides of the joint (from the beams) and, $V_{col\,t}$ and $V_{col\,b}$ are the shears applied to the top and bottom of the joint (from the columns). The h_b and h_c are the beam and column depths respectively, where $h'_b \approx 0.9h_b$ and $h'_c \approx 0.9h_c$. The h'_b and h'_c are approximate distance between the lines of action of the flexural compression found in the beams and columns on opposite sides of the joints.

Level 2 Joint Shear Design

Joint 3

Horizontal Joint Shear

Gravity induced joint shear:

$$V_{G+Qu,jh} = \frac{0.1 + 1.2 - \frac{1}{2}\left[27.0 + (-26.2)\right] \times 0.9 \times 1.2}{0.9 \times 1.6} = 0.60\ \text{kN}$$

As illustrated here, joint shear resulted from gravity loads is small. Consequently, gravity induced joint shear could be considered negligible in this instance.

Earthquake induced joint shear:

$$V_{E,jh} = \frac{37.4 + 65.4 - \frac{1}{2}(77.2 + 82.4) \text{ x } 0.9 \text{ x } 1.2}{0.9 \text{ x } 1.6} = 11.5\ \text{kN}$$

Limited ductility design requires

$\phi V_n = V_{jh} = V_{G+Qu,jh} + 2V_{E,jh}$

$\Rightarrow$ $V_{jh} = 0 + 2 \text{ x } 11.5$ (Gravity induced joint shear is considered negligible)

$= 23.0$ kN

Nominal shear stress in the joint

$$v_{jh} = \frac{V_{jh}}{b_c h_c} = \frac{23.0 \times 10^3}{190 \times 1200} = 0.10\ \text{MPa} < v_g = 0.45\sqrt{16} = 1.8\ \text{MPa}$$

Therefore OK

From section 11.4.5.2, since beams remain elastic (i.e. no hinging)

$$V_{sh} = \frac{V_{jh}}{\phi} - V_{mh}$$

where $V_{mh} = 0.5V_{jh} = 11.5$ kN

but need not be taken less than $V_{mh} = v_m b_c h_c$

where $v_m = (C_1 + C_2)v_{bm}$

and $C_1 = 33p_w \frac{f_y}{300}$

$p_w = 0.00297$ (for D12 @ 200 crs)

$\Rightarrow \quad C_1 = 0.098$

$C_2 = 1.0$ for simplicity

$$\Rightarrow \quad v_m = (0.098 + 1) \times 0.2\sqrt{f'_m}$$

$$= 0.22 \times \sqrt{16}$$

$$= 0.88 \text{ MPa}$$

Therefore $V_{mh} = 0.88 \times 0.19 \times 1.2 \times 10^3 = 200.6$ kN

Hence

$$V_{sh} = \frac{23.0}{0.75} - 200.6 < \text{ZERO}$$

Therefore NO horizontal joint steel is required (i.e. $A_{jh} = 0$). The horizontal shear is carried by the horizontal component of the diagonal strut across the joint.

Vertical Joint Shear

Earthquake induced joint shear:

$$V_{E,jv} = \frac{69.5 + 74.6 - \left(\frac{62.3 + 109.0}{2}\right) \times 0.9 \times 1.6}{0.9 \times 1.2}$$

$$= 19.2 \text{ kN}$$

$\phi V_n = V_{jv} = 2V_{E,jv}$ (Gravity induced joint shear is considered negligible in this instance)

$\Rightarrow \quad V_{jv} = 38.4$ kN

Nominal shear stress in the joint

$$v_{jv} = \frac{V_{jv}}{b_c h_b} = \frac{38.4 \times 10^3}{190 \times 1600} = 0.13 \text{ MPa} < v_g \quad \text{Therefore OK}$$

$$V_{sv} = \frac{V_{jv}}{\phi} - V_{mv}$$

where $V_{mv} = 0$ since potential plastic hinge regions are expected to form in the pier above and below the joint (see 11.4.6.2 of NZS 4230:2004).

Hence

$$V_{sv} = \frac{38.4}{0.75} - 0 = 51.2 \text{ kN}$$

and the total area of vertical joint shear reinforcement required:

$$A_{jv} = \frac{V_{sv}}{f_y} = \frac{51.2 \times 10^3}{300} \qquad \text{(Take } f_y = 300 \text{ MPa)}$$

$$= 170.7 \text{ mm}^2$$

Therefore, use 6-R6 to give $A_{jV} = 169.6 \text{ mm}^2$.

Joint 4

Horizontal Joint Shear

Earthquake induced joint shear:

$$V_{E,jh} = \frac{16.6 + 29.1 - \frac{1}{2} \text{ x } 82.4 \text{ x } 0.9 \text{ x } 0.8}{0.9 \text{ x } 1.6} = 11.1 \text{ kN}$$

Limited ductility design requires

$\phi V_n = V_{jh} = 2V_{E,jh}$ (Gravity induced joint shear is considered negligible in this instance)

$$\Rightarrow \quad V_{jh} = 2 \text{ x } 11.1$$

$$= 22.2 \text{ kN}$$

Nominal shear stress in the joint

$$v_{jh} = \frac{V_{jh}}{b_c h_c} = \frac{22.2 \times 10^3}{190 \times 800} = 0.15 \text{ MPa} \quad < v_g$$

From section 11.4.5.2, since beams remain elastic (i.e. no hinging)

$$V_{sh} = \frac{V_{jh}}{\phi} - V_{mh}$$

where $V_{mh} = 0.5V_{jh} = 11.1$ kN

but need not be taken less than $V_{mh} = v_m b_c h_c$

where $v_m = (C_1 + C_2) v_{bm}$

and $C_1 = 33 p_w \dfrac{f_y}{300}$

$p_w = 0.00297$ (for D12 @ 200 crs)

$\Rightarrow \quad C_1 = 0.098$

$C_2 = 1.0$ for simplicity

$$\Rightarrow \quad v_m = (0.098 + 1) \times 0.2\sqrt{f'_m}$$

$$= 0.22 \times \sqrt{16}$$

$$= 0.88 \text{ MPa}$$

Therefore $V_{mh} = 0.88 \times 0.19 \times 0.8 \times 10^3 = 133$ kN

Hence

$$V_{sh} = \frac{22.2}{0.75} - 133 < \text{ZERO}$$

Therefore NO horizontal joint steel is required (i.e. $A_{jh} = 0$). The horizontal shear is carried by the horizontal component of the diagonal strut across the joint.

<u>Vertical Joint Shear</u>

$$V_{E,jv} = \frac{73.7 - \left(\frac{27.7 + 48.5}{2}\right) \times 0.9 \times 1.6}{0.9 \times 0.8} = 26.2 \text{ kN}$$

$\phi V_n = V_{jv} = 2V_{E,jv}$ (Gravity induced joint shear is considered negligible in this instance)

$\Rightarrow \quad V_{jv} = 52.3$ kN

Nominal shear stress in the joint

$$v_{jv} = \frac{V_{jv}}{b_c h_b} = \frac{52.3 \times 10^3}{190 \times 1600} = 0.17 \text{ MPa} < v_g \qquad \text{Therefore OK}$$

$$V_{sv} = \frac{V_{jv}}{\phi} - V_{mv}$$

where $V_{mv} = 0$, see 11.4.6.2 of NZS 4230:2004.

Hence

$$V_{sv} = \frac{52.3}{0.75} - 0 = 69.7 \text{ kN}$$

Therefore, the total area of vertical joint shear reinforcement required:

$$A_{jv} = \frac{V_{sv}}{f_y} = \frac{69.7 \times 10^3}{300} \qquad (\text{Take } f_y = 300 \text{ MPa})$$

$$= 232.4 \text{ mm}^2$$

Therefore, use <u>4-R10</u> to give $A_{jv} = 314.2 \text{ mm}^2$.

Level 3 Joint Shear Design

A similar process to that above is required, but not tabulated herein, see Figure 18 for detailed.

3.8 Strut-and-tie Design of Wall with Opening

Figure 19(a) shows a three-storey concrete masonry wall with openings and loading conditions that resemble a design example of a reinforced concrete wall reported by Paulay and Priestley (1992). It is noted that designers may elect to consider a more sophisticated loading pattern, with horizontal loads apportioned within the wall based upon tributary areas, rather than the simple lumped horizontal forces shown in Figure 19(a).

The concrete masonry wall shown in Figure 19(a) is to be designed for the seismic lateral forces corresponding with an assumed ductility of ε = 2. The relatively small gravity loads are approximated by a number of forces at node points given in Figure 19(a), and the strut-and-tie model for the gravity loads is represented in Figure 19(b). Wall width should be 190 mm, and f'_m = 12 MPa. It is required to design the flexural and shear reinforcement for the wall.

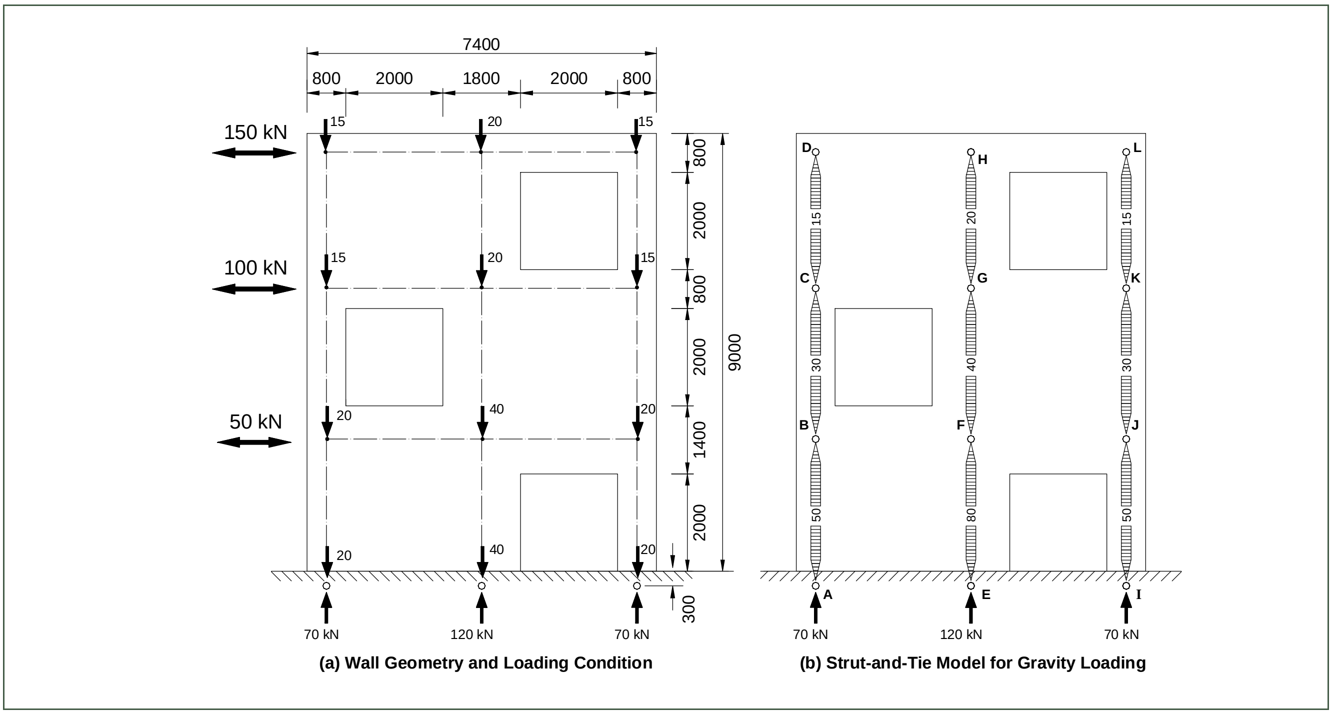

Figure 19: **Limited Ductile 3-storey Masonry Wall with Openings**

SOLUTION

Figures 20(a) and (b) show the strut-and-tie models for the squat wall with openings, corresponding to the seismic lateral forces being considered. For the purpose of limited ductile design, particular tension chords should be chosen to ensure yielding can best be accommodated. For example, members I-J and E-F in Figure 20(a) represent a good choice for this purpose.

Corresponding forces in other members should be determined and hence reinforcement provided so as to ensure that no yielding in other ties can occur. As these members carry only tension, yielding with cyclic displacements may lead to unacceptable cumulative elongations. Such elongations would impose significant relative secondary displacement on the small piers adjacent to openings, particularly those at I-J and A-B. The resulting bending moment and shear forces, although secondary, may eventually reduce the capacity of these vital struts.

In order to ensure that plastic hinges form inside the 1st storey vertical members, the quantity of reinforcement in the 2nd and 3rd storey vertical members should be sufficient to ensure that yielding does not occur in these

members. Consequently, a simplified procedure is adopted in this example to design the vertical tie members above 1st storey for 50% more tension force than design levels.

From the given lateral forces the total overturning moment at 300 mm below the wall base is:

$$M^* = 150 \times (8.6 + 0.3) + 100 \times (5.8 + 0.3) + 50 \times (2.7 + 0.3)$$

$$= 2095 \text{ kNm}$$

Whilst the use of strut-and-tie analysis is specifically endorsed in section 7.4.8.1 of NZS 4230:2004, no advice is given in section 3.4.7 for an appropriate ϕ value to be used in conjunction with the analysis. In section 2.3.2.2(h) of NZS 3101:2006 a value of $\phi = 0.75$ is prescribed. This corresponds to the ϕ factor used for shear and torsion, which is consistent with the strut-and-tie procedure. Consequently, $\phi = 0.75$ is adopted here for use in strut-and-tie analysis of concrete masonry structures.

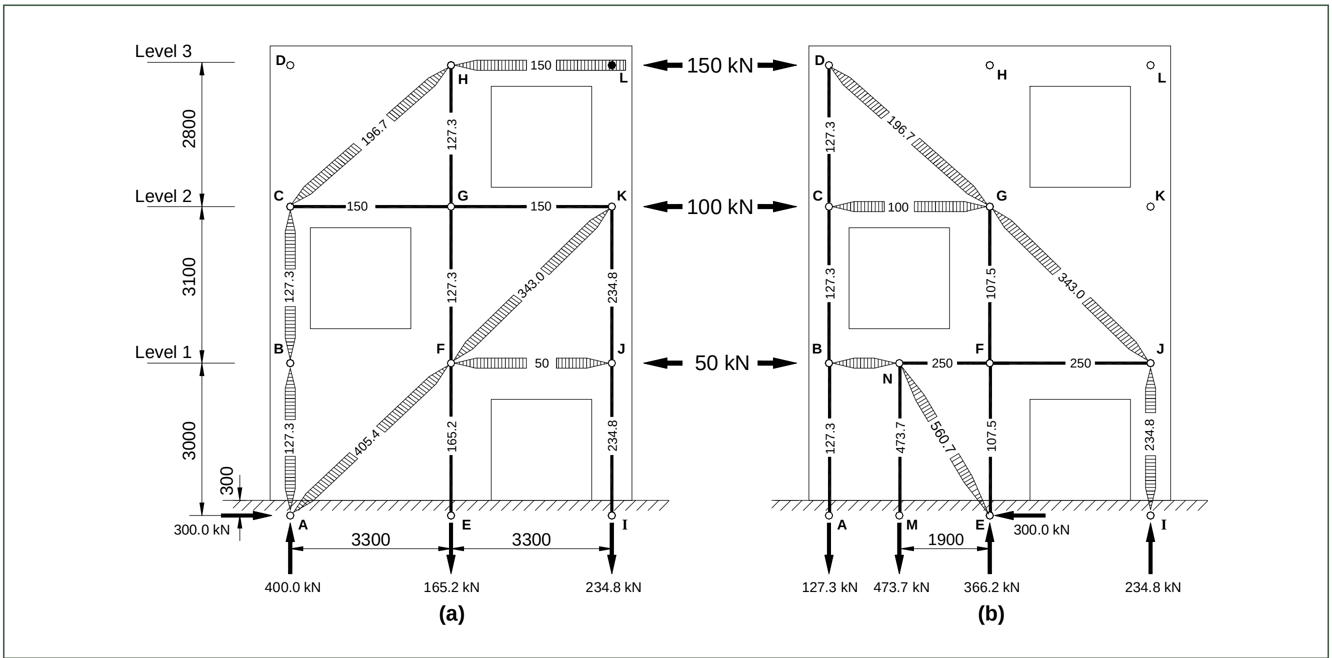

Figure 20: **Strut-and-Tie Models for Masonry Wall (seismic loading only)**

Design of Tension Reinforcement in Vertical Members

The area of tension reinforcement required in vertical ties, after considering the effect of axial loads, can be evaluated as follows:

$$\phi\left(A_{si} f_y + N_n\right) = T_i$$

$$\phi\left(A_{si} f_y + \frac{N_i^*}{\phi}\right) = T_i$$

Therefore

$$\phi A_{si} f_y = T_i - N_i^* \quad ^{(8)}$$

Figure 21 shows the strut-and-tie model for the squat wall when both seismic and gravity loads are considered.

8 Paulay and Priestley (1992) adopted the procedure of $\phi A_{si} f_y = T_i - \phi N_i^*$, as this would result in a more conservative design.

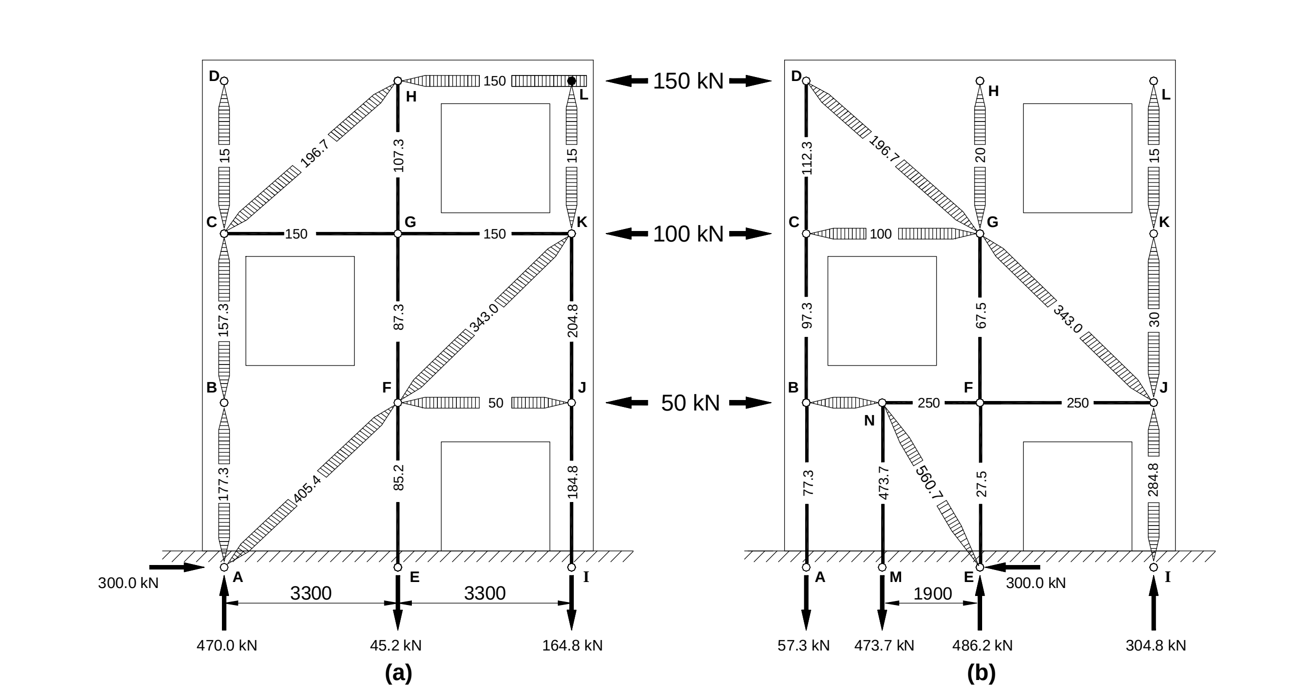

Figure 21: **Strut-and-Tie Models for Masonry Wall (Seismic and Gravity Loads)**

1st Storey Vertical Members

Consider earthquake $\overleftarrow{V}_E$ **as in Figure 21(a)**

Tie I-J $\phi A_{IJ} f_y = 184.8$ kN

Therefore $$\phi A_{IJ} = \frac{184.8 \times 10^3}{0.75 \times 300}$$ (taking $f_y = 300$ MPa)

$= 821.3$ mm^2

Try 4-D16 $A_s = 804.2$ mm^2 (about 2% shortfall)

Tie E-F $\phi\, A_{EF} f_y = 85.2$ kN

Therefore $$A_{EF} = \frac{85.2 \times 10^3}{0.75 \times 300}$$

$= 378.7$ mm^2

Clause 7.4.5.1 of the standard requires minimum longitudinal reinforcement of D12 @ 400 crs within the potential plastic hinge zone. Consequently, adopt 5-D12 for Member E-F to give $A_s = 565.5$ mm^2.

Check moment capacity at wall base:

Tension forces provided:

$T_{IJ} = 804.2 \times 300 = 241.3$ kN

$T_{EF} = 565.5 \times 300 = 169.6$ kN

Therefore, total compression force at Node A, including gravity load:

$$C_m = T_{IJ} + T_{EF} + N_n$$

$$= 241.3 + 169.6 + \frac{260}{0.75}$$

$$= 757.6 \text{ kN}$$

Theoretical depth of neutral axis:

$$c = \frac{C_m}{0.85 \times 0.85 \times f'_m \times 190}$$

$$= \frac{757.6 \times 10^3}{0.85 \times 0.85 \times 12 \times 190}$$

$= 459.9$ mm $\approx 0.100L_w$ where $L_w = 800 + 2000 + 1800 = 4600$ mm

$< 0.2L_w$ (see clause 7.4.6.1 of NZS 4230:2004)

Moment capacity about the centre of the structure:

$M_n = (T_{IJ} + C_m) \times 3.3 = (241.3 + 757.6) \times 3.3$

$= 3296.4$ kNm

Therefore $\phi M_n = 0.75 \times 3395.7$

$= 2472.3$ kNm $> M^*$

Consider earthquake $\overrightarrow{V_E}$ as in Figure 21(b)

<u>Tie A-B</u> $\phi A_{AB} f_y = 77.3$ kN

Therefore $A_{AB} f_y = \dfrac{77.3 \times 10^3}{0.75 \times 300}$ (taking $f_y = 300$ MPa)

$= 341.6 \text{ mm}^2$

Try <u>4-D12</u> $A_s = 452.4 \text{ mm}^2$

<u>Tie M-N</u> $\phi A_{MN} f_y = 473.7$ kN

Therefore $A_{MN} = \dfrac{473.7 \times 10^3}{0.75 \times 300}$ (taking $f_y = 300$ MPa)

$= 2105.3 \text{ mm}^2$

Try <u>8-D16 and 2-D20</u> $A_s = 2236.8 \text{ mm}^2$ (Note that D20 is the maximum bar size allowed for 190 mm wide masonry wall)

<u>Tie E-F</u> Use <u>5-D12</u> because member force would be critical when earthquake force acting in $\overleftarrow{V_E}$ direction, i.e. $A_s = 565 \text{ mm}^2$. Refer to Figure 22 for details.

<u>Check moment capacity at wall base:</u>

Tension forces provided:

T_{AB} = 452.4 x 300 = 135.7 kN

T_{MN} = 2236.8 x 300 = 671.0 kN

T_{EF} = 565.5 x 300 = 169.6 kN

Therefore, total compression force at Node I, including gravity load:

$$C_m = T_{AB} + (T_{MN} - T_{MN}) + T_{EF} + N_n$$

$$= 135.7 + (671.0 - 671.0) + 169.6 + \frac{260}{0.75}$$

$$= 652.1 \text{ kN}$$

Note that in the above calculation, it is recognised that the vertical component of strut E-N matches the force in tie M-N.

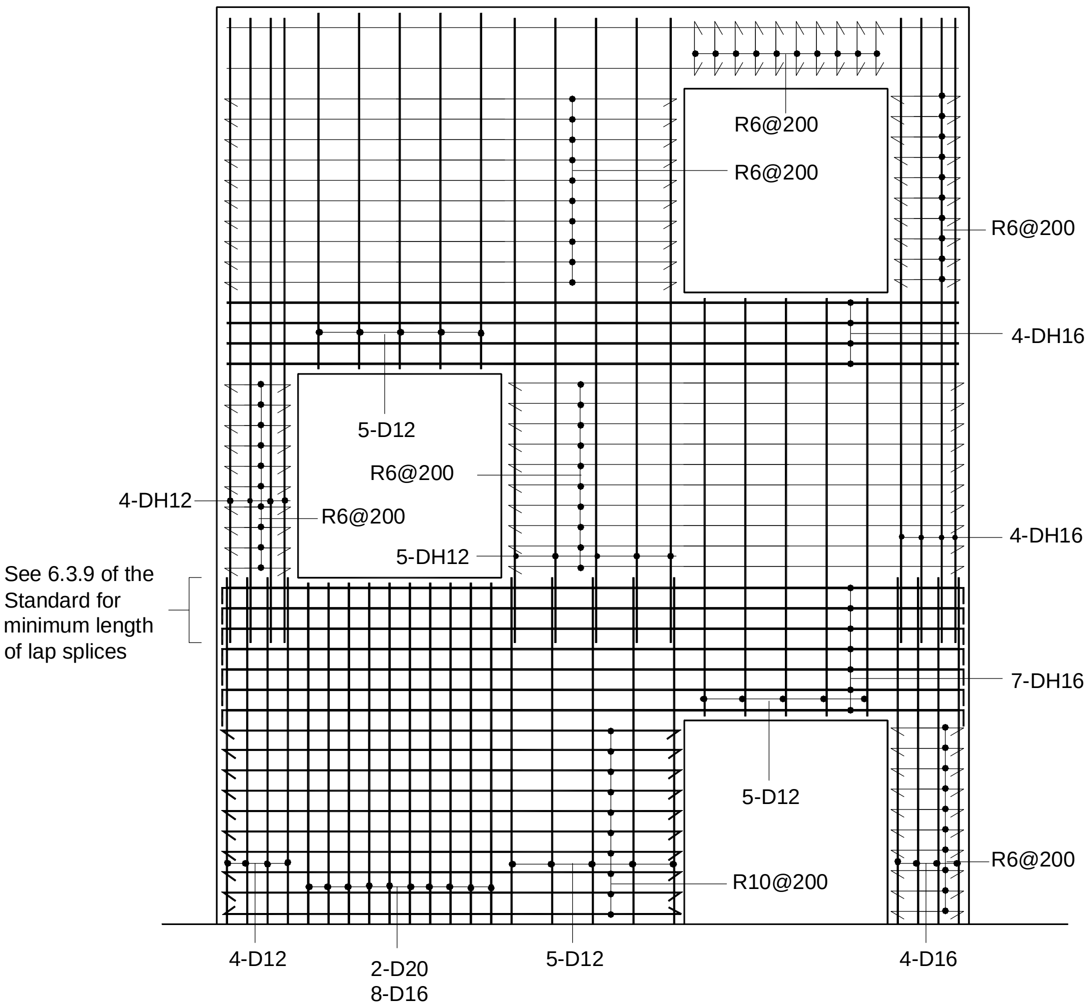

Figure 22: **Reinforcement for Design Example 3.8**

Theoretical depth of neutral axis:

$$c = \frac{C_m}{0.85 \times 0.85 \times f'_m \times 190}$$

$$= \frac{652.1 \times 10^3}{0.85 \times 0.85 \times 12 \times 190}$$

$$= 395.8 \text{ mm} \quad \approx 0.086L_w$$

Moment capacity about the centre of the structure:

$$M_n = (T_{AB} + C_m) \text{ x } 3.3 + T_{MN} \text{ x } 1.9$$

$$= (135.7 + 652.1) \text{ x } 3.3 + 671.0 \text{ x } 1.9$$

$$= 3874.6 \text{ kNm}$$

Therefore $\phi M_n = 0.75 \text{ x } 3874.6$

$= 2906 \text{ kNm} > M^*$

2nd and 3rd Storey Vertical Members

To avoid the formation of plastic hinges, the amount of reinforcement in the 2nd and 3rd storey vertical members should be sufficient to ensure that yielding does not occur in these members. Hence, the 2nd and 3rd storey vertical members are intentionally designed for 50% higher tension forces than the design level tension forces.

Consider earthquake $\overleftarrow{V}_E$ as in Figure 21(a)

Tie J-L $\qquad \phi A_{JK} f_y = 1.5 \text{ x } 204.8$

$= 307.2$ kN

For tie J-L, the force in tie J-K is critical. Therefore, the design of tie K-L will match that of tie J-K.

Therefore $\quad A_{JK} = \dfrac{307.2 \times 10^3}{0.75 \times 500}$ (take $f_y = 500$ MPa)

$= 819.2 \text{ mm}^2$

Try 4-DH16 $\quad A_s = 804.2 \text{ mm}^2$ (about 2% shortfall)

(Note that DH16 is the maximum bar size allowed in Table 1)

Tie F-H $\qquad \phi A_{GH} f_y = 1.5 \text{ x } 107.3$

$= 161.0$ Kn

For tie F-H, the force in tie G-H is critical. Therefore, the design of tie F-G will match that of tie G-H.

Therefore $\quad A_{GH} = \dfrac{161.0 \times 10^3}{0.75 \times 500}$ (take $f_y = 500$ MPa)

$= 429.3 \text{ mm}^2$

Try 5-DH12 $\quad A_s = 565.5 \text{ mm}^2$

Consider earthquake $\overrightarrow{V_E}$ as in Figure 21(b)

Tie B-D $\phi A_{CD} f_y = 1.5 \times 112.3$

$= 168.5$ kN

For tie B-D, the force in tie C-D is critical. Therefore, the design of tie B-C will match that of tie C-D.

Therefore $$A_{CD} = \frac{168.5 \times 10^3}{0.75 \times 500} \quad \text{(take } f_y = 500 \text{ MPa)}$$

$$= 449.3 \text{ mm}^2$$

Try 4-DH12 $A_s = 452.4 \text{ mm}^2$

Tie F-H Use 5-DH12 because member force would be critical when earthquake force acting in $\overleftarrow{V_E}$ direction, i.e. $A_s = 565.5 \text{ mm}^2$. Refer to Figure 22 for details.

Design of Tension Reinforcement in Horizontal Members

In section 3.7.3.3 of NZS 4230:2004, there are two equations given that permit a simplified capacity design approach to be used. However, in this example it has been necessary to place a significantly larger quantity of vertical reinforcement than required (i.e. member E-F), in order to satisfy spacing criteria.

This has resulted in a concern about relying upon these simplified expressions and instead a full capacity design is conducted below to establish the appropriate horizontal design forces.

To estimate the maximum tension force in horizontal ties, the flexural overstrength at wall base, M_o, needs to be calculated:

$M_o = 1.25 M_{n,provided}$

Consider earthquake $\overleftarrow{V_E}$ as in Figure 21(a)

$M_{n,provided} = 3296.4$ kNm

The overstrength value, $\phi_{o,w}$, is calculated as follows:

$$\phi_{o,w} = \frac{M_o}{M^*} = \frac{1.25 M_{n,provided}}{M^*}$$

$$= \frac{1.25 \times 3296.4}{2095}$$

$$= 1.97$$

Dynamic magnification factor:

For up to 6 storeys $$\omega_v = 0.9 + \frac{n}{10}$$

$$= 0.9 + \frac{2}{10}$$

$$= 1.1$$

Hence, the design force for Member C-G-K is calculated as follow:

T_{CK} = 1.1 x 1.97 x 150

= 325.1 kN

Therefore

$\phi A_{ck} f_y$ = 325.1 kN

$$A_{ck} = \frac{325.1 \times 10^3}{1.0 \times 500} \qquad \phi = 1.0 \text{ (see 3.4.7) and take } f_y = 500 \text{ MPa}$$

= 650.2 mm^2

Try 4-DH16 A_s = 804 mm^2

Consider earthquake $\overrightarrow{V_E}$ as in Figure 21(b)

$M_{n,provided}$ = 3874.6 kNm

The overstrength value, $\phi_{o,w}$, is calculated as follow:

$$\phi_{o,w} = \frac{M_o}{M^*} = \frac{1.25 M_{n,provided}}{M^*}$$

$$= \frac{1.25 \times 3874.6}{2095}$$

= 2.31 > $\phi_{o,w}$ = 1.97 when considering $\overleftarrow{V_E}$

Dynamic magnification factor:

For up to 6 storeys $\omega_V = 1.1$

Hence, the design force for Member N-F-J is calculated as follow:

T_{NJ} = 1.1 x 2.31 x 250 = 635.3 kN

Therefore

$\phi A_{NJ} f_y$ = 635.3 kN

$$A_{NJ} = \frac{635.3 \times 10^3}{1.0 \times 500} \quad \text{(take } f_y = 500 \text{ MPa)}$$

= 1270.6 mm^2

Try 7-DH16 A_s = 1407.4 mm^2

Design of Shear Reinforcement

It is assumed that shear forces are to be resisted by the bigger wall elements adjacent to openings, such that only these elements require design of shear reinforcement. For other part of the wall structure, it is only required to satisfy p_{min} = 0.07%, i.e. use R6 @ 200 crs.

As V_G^* and V_{Qu}^* are typically negligible, therefore:

$$\phi V_n \geq \omega_v \phi_{o,w} V_E^* \qquad \text{where } \phi = 1.0 \text{ (3.4.7 of NZS 4230:2004)}$$

Shear Design, 1st Storey

$$V_E^* = 300 \text{ kN}$$

Therefore $V_n = \dfrac{1.1 \times 2.31 \times 300}{1.0}$

$= 762.3$ kN

Check shear stress, $b_w = 190$ mm, $d = 0.8 \times 4600 = 3680$ mm

$$v_n = \frac{762.3 \times 10^3}{190 \times 3680} = 1.09 \text{ MPa} < v_g = 1.50 \text{ MPa for } f'_m = 12 \text{ MPa}$$

From Section 10.3 of NZS 4230:2004:

$$v_n = v_m + v_p + v_s$$

Shear stress carried by $v_m = (C_1 + C_2)v_{bm}$

where $C_1 = 33 p_w \dfrac{f_y}{300}$

note that $p_w = \dfrac{9\text{bars} \times D12 + 8\text{bars} \times D16 + 2\text{bars} \times D20}{b_w d}$

$= \dfrac{3254.7}{190 \times 0.8 \times 4600}$

$= 0.0046$

Therefore $C_1 = 0.15$

and $C_2 = 0.42 \times [4 - 1.75 \times (3400/4600)]$

$= 1.14$

$\Rightarrow \quad v_m = (0.15 + 1.14) \times v_{bm}$

$= 1.29 \times 0.50$ note that $v_{bm} = 0.50$ MPa for $\varepsilon = 2$

$= 0.67$ MPa

Therefore the shear reinforcement required:

$v_s = v_n - v_m - v_p$ (take $v_p = 0$ for simplicity)

$\Rightarrow \quad v_s = 1.09 ï\ 0.67 - 0$

$= 0.42$ MPa

$$v_s = C_3 \frac{A_v f_y}{b_w s}$$ note that $C_3 = 0.8$ for masonry walls

$$\Rightarrow \quad 0.42 = 0.8 \times \frac{A_v \times 300}{190 \times 200}$$ (try $f_y = 300$ MPa and s = 200 mm)

$A_v = 66.5\ mm^2$

Therefore, use R10 @ 200 crs (78.5 mm^2) and $p = \frac{78.5}{190 \times 200} = 0.2\%$.

Shear Design, 2nd Storey

$V_E^* = 250$ kN

Therefore V_n = 1.1 x 2.31 x 250

= 635.3 kN

Check shear stress, b_w = 190 mm, d = 0.8 x 4600 = 3680 mm

$$v_n = \frac{635.3 \times 10^3}{190 \times 3680} = 0.91\text{ MPa} < v_g = 1.50\text{ MPa}$$

Shear stress carried by $v_m = (C_1 + C_2)v_{bm}$

where $C_1 = 33 p_w \frac{f_y}{300}$

$$= 33 \times \frac{5\,\text{bars} \times DH12 + 4\,\text{bars} \times DH16}{b_w d} \times \frac{500}{300} + 33 \times \frac{5\,\text{bars} \times D12}{b_w d} \times \frac{300}{300}$$

= 0.10 + 0.03

= 0.13

and $C_2 = 0.42 \times \left[4 - 1.75 \times \left(\frac{4200}{4600}\right)\right]$

= 1.01

$$\Rightarrow \quad v_m = (0.13 + 1.01) \times v_{bm}$$

$= 1.14 \times v_{bm}$ (v_{bm} = 0.70 MPa since outside plastic hinge region)

= 1.14 x 0.70

= 0.80 MPa

Therefore the shear reinforcement required:

$v_s = v_n - v_m - v_p$ (take $v_p = 0$ for simplicity)

$\Rightarrow$ v_s = 0.91 ï 0.80 - 0

= 0.11 MPa

$$v_s = C_3 \frac{A_v f_y}{b_w s}$$ where C_3 = 0.8 for masonry walls

$$\Rightarrow \quad 0.11 = 0.8 \times \frac{A_v \times 300}{190 \times 200}$$ (try f_y = 300 MPa and s = 200 mm)

$A_v = 17.5\ mm^2$

Therefore, use R6 @ 200 crs (28.3 mm^2) and $p = \frac{28.3}{190 \times 200} = 0.07\%$. Note that p = 0.07% is the minimum reinforcement area required by 7.3.4.3 of NZS 4230:2004.

Shear Design, 3rd Storey

$V_E^* = 150$ kN

therefore $V_n = 1.1 \times 2.31 \times 150$

$= 381.2$ kN

Check shear stress, b_w = 190 mm, d = 0.8 x 4600 = 3680 mm

$$v_n = \frac{381.2 \times 10^3}{190 \times 3680} = 0.54 \text{ MPa} \quad < v_g$$

Shear stress carried by $v_m = (C_1 + C_2)v_{bm}$

where $C_1 = 33p_w \frac{f_y}{300} = 33 \times \frac{9 \text{ bars} \times DH12}{b_w d} \times \frac{500}{300} + 33 \times \frac{5 \text{ bars} \times D12}{b_w d} \times \frac{300}{300}$

$= 0.08 + 0.03$

$= 0.11$

and $C_2 = 0.42 \times \left[4 - 1.75 \times \left(\frac{3600}{4600}\right)\right]$

$= 1.10$

$\Rightarrow \quad v_m = (0.11 + 1.10) \times v_{bm}$ (v_{bm} = 0.70 MPa outside plastic hinge region)

$= 1.21 \times 0.70$

$= 0.85 \text{ MPa} > v_n$

Because $v_m > v_n$, the shear reinforcement needed in the 3rd storey pier is governed by the minimum reinforcement area required by clause 7.3.4.3, i.e. 0.07% of the gross cross-sectional area. Therefore, shear reinforcement in the 3rd storey pier can be reduced to R6 @ 200 crs.

4.0 Prestressed Masonry

A new addition to NZS 4230 is the inclusion of Appendix A related to the design of prestressed concrete masonry. As noted in the commentary, this section is primarily for application to wall components, but its use for other component types is not precluded.

Design information for unbonded post-tensioning is presented below.

This form of prestressing is recommended as it minimises structural damage and results in structures that exhibit little or no permanent horizontal deformation following earthquake excitation. It is noted that the provided information is more comprehensive than will be required for most conventional designs, and is included as background for the following example.

For additional information refer to research conducted by Laursen and Ingham at the University of Auckland[9,10].

4.1 Limit States

The flexural design procedure presented here is based on Limit State Design, as outlined by AS/NZS 1170.0:2002, which identifies two limit states, namely the Serviceability limit state and the Ultimate limit state.

The flexural serviceability limit state for prestressed masonry is concerned with flexural strength, stiffness and deflections. The following flexural states represent the limiting flexural moments for a wall to remain elastic for uncracked and cracked sections.

- *First Cracking:* This limit state corresponds to the state when the extreme fibre of the wall decompresses (the tensile strength of concrete masonry is disregarded)
- *Maximum Serviceability moment:* At this cracked section state, the compressive stress in the extreme compression fibre has reached its elastic limit set out by the standard as a stress limitation. Reinforcement and concrete masonry remain elastic in this state.

The flexural ultimate limit state for prestressed masonry is primarily concerned with flexural strength. Additionally for ductility purposes, overstrength, stiffness and deflections should be considered:

- *Nominal strength:* The nominal strength according to NZS 4230:2004 is per definition achieved when the concrete masonry fails in compression at the strain, ε_u, equals 0.003.
- *Overstrength*: This strength corresponds to the maximum moment strength developed by the wall, taking into account stress increase, yield and strain hardening of the prestressing tendons. At this stage, large deformations are expected and the maximum concrete masonry strain is likely to have surpassed 0.003. Past the maximum wall strength, the wall resistance gradually degrades until failure.

All of the above limit states generally need to be evaluated both immediately after prestress transfer and after long term losses.

4.2 Flexural Response of Cantilever Walls

This section considers the flexural design of prestressed concrete masonry cantilever walls with unbonded prestressing tendons, where the lateral force is assumed to be acting at the top of the wall or at some effective height h_e, refer to Figure 23.

For other structural shapes and loading configurations, the formulae should be modified accordingly. Note that the term "tendon" in the following sections refer to both prestressing strands and bars.

The applied forces and loads represented by the symbols V, M, N and P used in the following equations are all factored loads calculated according to the applicable limit state as defined in the AS/NZS 1170.0:2002. The axial force N is due to dead and live loads, P is the prestressing force (initial force after anchor lock-off or force after all long term losses), and V is the applied lateral force due to lateral actions. It is assumed that moment M only arises from lateral forces V, i.e. permanent loads and prestressing do not introduce permanent moment in the wall.

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9 Laursen, P. T. (2002) "Seismic Analysis and Performance of Post-Tensioned Concrete Masonry Walls", Doctoral Thesis, University of Auckland, 281pp.

10 Laursen, P. T., and Ingham, J. M. (1999) "Design of Prestressed Concrete Masonry Walls", Journal of the Structural Engineering Society of New Zealand, 12, 2, 21-39.

Figure 23 shows the various definitions of wall dimensions and forces.

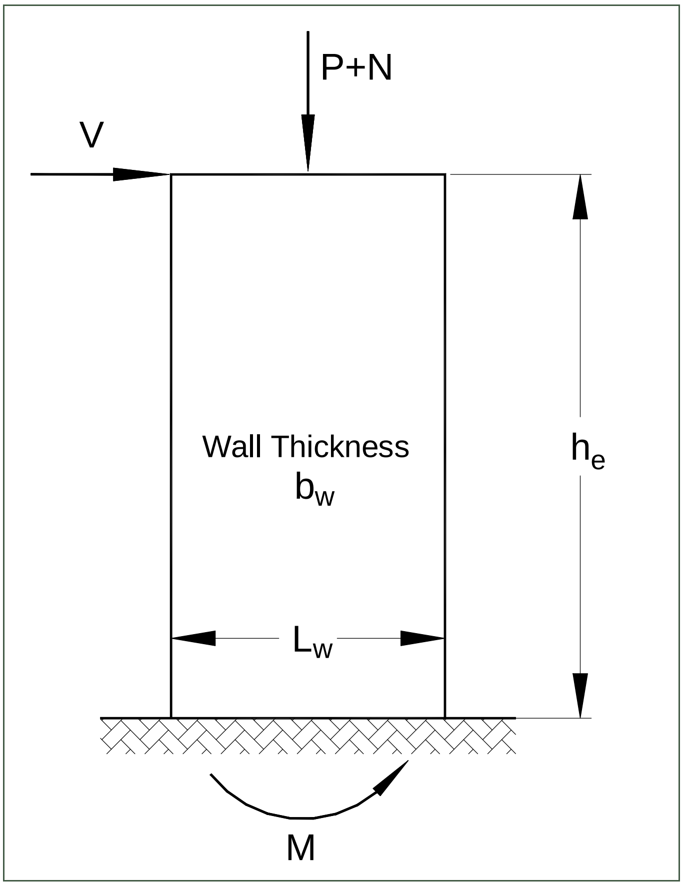

Figure 23: **Definition of Wall**

It is assumed for the flexural calculations that plane sections remain plane, i.e. a linear strain distribution across the wall length. This assumption enables analytical calculation of strength, stiffness and displacement, and implies distributed cracking up the wall height.

From laboratory wall tests it was observed that PCM wall flexural response was primarily due to rocking where a crack opened at the base, and that distributed flexural cracking did not develop[9]. This type of rocking behaviour is a feature of prestressing with unbonded tendons.

Despite this discrepancy between theory and observation, it appears that the assumption of plane section response and distributed wall cracks results in sufficiently accurate design rules.

4.2.1 First Cracking

The moment corresponding to first cracking M_{cr} may be evaluated by Eqn. 18. The formula is based on the flexural state at which one wall end decompresses and the other end compresses to a stress of twice the average masonry stress f_m:

$$M_{cr} = \frac{f_m b_w L_w^2}{6} = \frac{(P+N)L_w}{6}, \quad f_m = \frac{P+N}{L_w b_w} \qquad [18]$$

$$V_{cr} = \frac{M_{cr}}{h_e} \qquad [19]$$

where b_w is the wall thickness, L_w is the wall length, V_{cr} is the applied force at the top of the wall corresponding to the 1st cracking moment M_{cr} and h_e is the effective wall height.

The deflection of the top of the wall d_{cr} at V_{cr} should be based on the concrete masonry wall elastic properties and consists of a component due to shear deformation $d_{cr,sh}$ and a component due to flexure $d_{cr,fl}$:

$$d_{cr} = d_{cr,fl} + d_{cr,sh} = \frac{2}{3}\frac{h_e^2\,(P+N)}{E_m L_w^2 b_w} + \frac{2}{5}\frac{(1+\nu)(P+N)}{E_m b_w} \qquad [20]$$

where Poisson's ratio may be taken as $\nu = 0.2$. It should be noted that the shear deformation component $d_{cr,sh}$ can be of significant magnitude for squat walls under serviceability loads, whereas for the ultimate limit state it becomes increasingly insignificant.

The curvature at 1st cracking can be calculated as follows:

$$\phi_{cr} = \frac{2(P+N)}{E_m L_w^2 b_w} \qquad [21]$$

4.2.2 Maximum Serviceability Moment

Typically at the serviceability limit state, the applied lateral force has surpassed that necessary to initiate cracking at the base of the wall. The serviceability moment is limited by M_e which occurs when the stress in the extreme compression fibre at the base of the wall has reached kf'_m, as shown in Figure 24. For prestressed concrete masonry, k (symbol adopted in this manual) is set out in Table A.1 of NZS 4230:2004, which is reproduced from Table 16.1 of NZS 3101:1995, with k typically ranging between 0.4 and 0.6, dependent on load category. In NZS 3101:2006 this criteria was modified, where clause 19.3.3.5.1(b) prescribes an upper limit of k = 0.6.

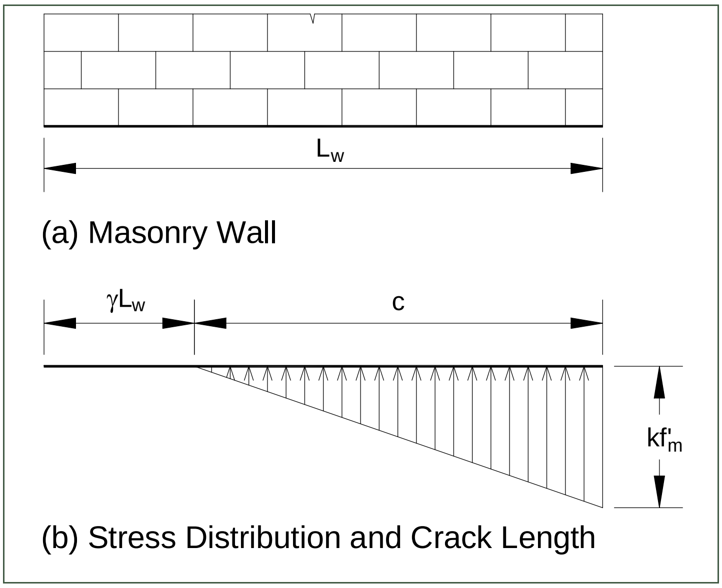

Figure 24: **Maximum Serviceability Moment**

It is noted that Eqn. 22 must be satisfied before use of the equations relating to the maximum serviceability moment can be applied, though this requirement is generally fulfilled.

$$kf'_m > 2f_m \qquad [22]$$

The masonry is assumed to remain linearly elastic, hence the masonry strain ε_{ms} corresponding to kf'_m can be found from:

$$\varepsilon_{ms} = \frac{kf'_m}{E_m} \qquad [23]$$

By adopting k = 0.55 from load category IV (infrequent transient loads), it may be shown that the maximum serviceability moment can be calculated as[9]:

$$M_e = \frac{f_m}{6}\left(3 - \frac{4f_m}{kf'_m}\right)L_w^2 b_w = f_m\left(0.5 - 1.2\frac{f_m}{f'_m}\right)L_w^2 b_w = V_e h_e \qquad [24]$$

where V_e is the corresponding lateral force. The corresponding curvature at the wall base, ϕ_e, is:

$$\phi_e = \frac{(kf'_m)^2}{2f_m E_m L_w} = 0.15\frac{{f'_m}^2}{f_m E_m L_w} \qquad [25]$$

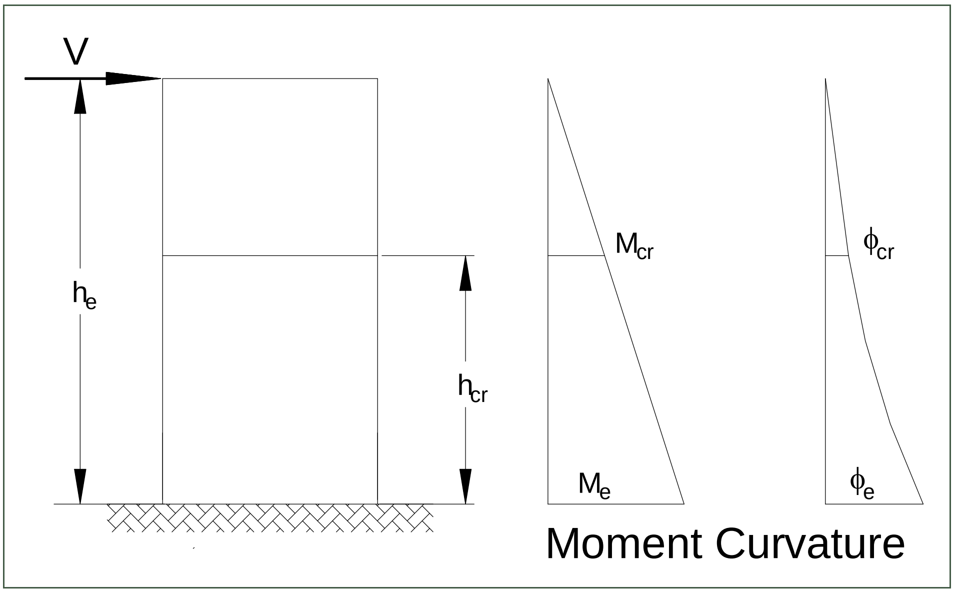

Figure 25: **Curvature Distribution at Maximum Serviceability Moment**

Figure 25 shows the variation of moment and curvature along the height of the wall at the maximum serviceability moment, assuming plane section response. The curvature varies from ϕ_e at the base to ϕ_{cr} at the height, h_{cr}, at which the 1st cracking occurs. Between the heights h_{cr} and h_e the curvature varies linearly between ϕ_{cr} and zero. It can be shown that the curvature varies linearly with the non-dimensional crack length, γ, as defined in Figure 24. Eqn. 26 defines the non-dimensional crack length at the base of the wall at the maximum serviceability moment, again assuming k = 0.55:

$$\gamma_e = 1 - \frac{2f_m}{kf'_m} = 1 - 3.6\frac{f_m}{f'_m} \qquad [26]$$

Eqn. 27 defines the resulting cracked wall height.

$$h_{cr} = h_e\left(\frac{M_e - M_{cr}}{M_e}\right) \qquad [27]$$

The total displacement d_e of the top of the wall can then be calculated by integration along the wall height with the following result:

$$d_e = d_{e,fl} + d_{e,sh} \qquad [28]$$

$$d_{e,fl} = \frac{2f_m h_{cr}}{E_m L_w \gamma_e}\left[(h_e - h_{cr})\left(\frac{\gamma_e}{1-\gamma_e}\right) + \frac{h_{cr}}{\gamma_e}\left(\frac{\gamma_e}{1-\gamma_e} + \ln\left|1-\gamma_e\right|\right)\right] + \frac{\phi_{cr}}{3}(h_e - h_{cr})^2 \qquad [29]$$

which may be approximated assuming k = 0.55 as:

$$d_{e,fl} = \left(0.30 - 0.029\frac{f_m}{f'_m}\right)\frac{f'_m h_e^2}{E_m L_w}$$

and

$$d_{e,sh} = \frac{12(1+\nu)h_e}{5E_m L_w b_w} V_e \qquad [30]$$

In Eqns. 29 and 30, $d_{e,fl}$ and $d_{e,sh}$ represent the flexural and shear deformations, respectively. At this flexural state, it is assumed that the relatively small deformations of the wall do not result in significant tendon force increase or migration of the tendon force eccentricity.

4.2.3 Nominal Strength

At the ultimate limit state, an equivalent rectangular stress block is assumed with a stress of 0.85 f'_m (α= 0.85) and an extreme fibre strain of ε_u = 0.003, corresponding to the definition of nominal strength in NZS 4230:2004 for unconfined concrete masonry. For confined masonry NZS 4230:2004 recommends using an average stress of 0.9Kf'_m (α = 0.9K with f'_m based on unconfined prism strength) and ε_u = 0.008. The corresponding moment M_n and lateral force V_f can be evaluated by simple equilibrium, as shown in Figure 26, with the following equation:

$$M_n = (P + \Delta P)\left(\frac{L_w}{2} + e_t - \frac{a}{2}\right) + N\left(\frac{L_w}{2} - \frac{a}{2}\right) = V_f h_e \qquad [31]$$

where a is the length of the equivalent ultimate compression block given by:

$$a = \frac{P + \Delta P + N}{\alpha f'_m b_w} \qquad [32]$$

In these equations, ΔP accounts for the increase in tendon force that arises from the flexural deformation and e_t accounts for the associated tendon force eccentricity. Both ΔP and e_t may initially be assumed to equal zero for simple use. This approach is similar to the method used in NZS 3101:2006. A better estimate of the nominal strength may be obtained from Eqn 31, when taking into account the tendon force increase ΔP and the associated tendon force eccentricity e_t.

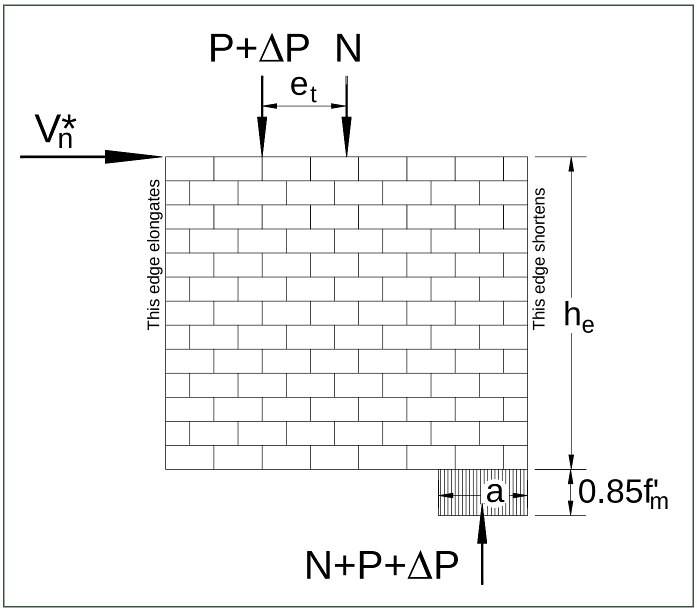

Figure 26: **Wall Equilibrium at Nominal Flexural Strength**

It is observed from Figure 26 that there is moment reversal near the top of the wall due to e_t which results in reversal of curvature. This effect is not taken into account below when calculating wall deformations because it has a negligible effect on the predicted wall behaviour at nominal flexural strength.

The total lateral displacement, d_n, is given by the sum of the flexural displacement, d_{nfl} , and shear displacement, d_{nsh}, corresponding to M_n, and may be evaluated using Eqn. 33:

$$d_n = d_{nfl} + d_{nsh} \text{ where} \qquad [33]$$

Unconfined: $$d_{nfl} = (2.30\xi_n^2 - 1.38\xi_n + 0.856)\frac{f'_m h_e^2}{E_m L_w} \qquad [34]$$

Confined: $$d_{nfl} = (7.63\xi_n^2 - 5.40\xi_n + 1.69)\frac{f'_m h_e^2}{E_m L_w} \qquad [35]$$

$$d_{n,sh} = \frac{12(1+\nu)h_e}{5E_m L_w b_w} V_f \qquad [36]$$

$$\xi_n = \frac{P + \Delta P + N}{f'_m L_w b_w} \qquad [37]$$

Eqns. 34 and 35 were developed using numerical integration and curve fitting, and are thus of an approximate nature, and are valid for axial load ratios, ξ_n, of 0.05 to 0.25. The extreme fibre strain was taken as ε_u = 0.003 for unconfined concrete masonry and 0.008 for confined concrete masonry. Detailed information on derivation of these equations may be found in Laursen[9].

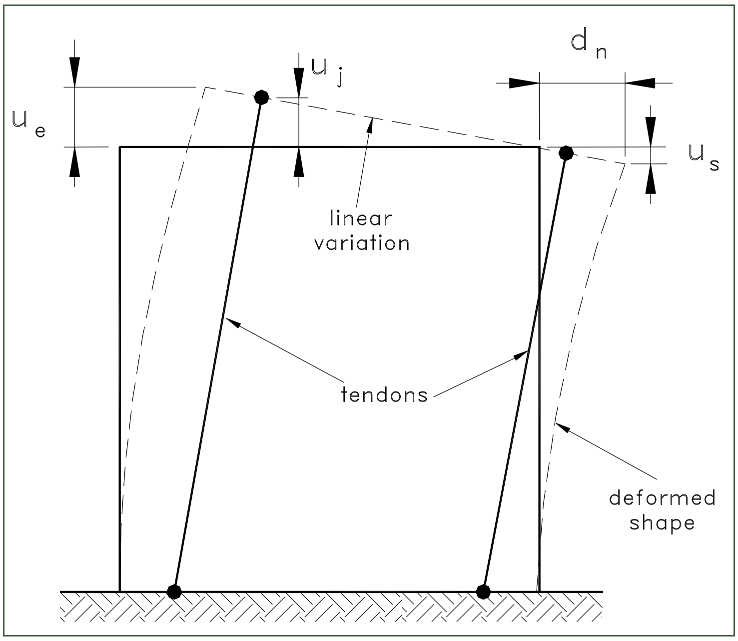

Figure 27: **Wall Deformation at Nominal Flexural Strength**

The total tendon force increase ΔP at ε_u of 0.003 (or 0.008) is difficult to evaluate for pre-stressed walls with unbonded tendons because the tendon stress increase depends on the deformation of the entire wall between points of anchorage. However, the force increase (or decrease) in each tendon in the wall cross section may be evaluated based on the estimated wall end elongation, u_e, (tension end) and shortening (compression end), u_s, assuming a linear variation of vertical deformation across the wall top as shown in Figure 27. The following equations were established for unconfined and confined concrete masonry[9]:

Unconfined: $$u_e = (4.01\xi_n^2 - 2.37\xi_n + 0.835)\frac{f'_m h_e}{E_m} \quad [38]$$

$$u_s = (3.36\xi_n^2 - 2.12\xi_n - 0.073)\frac{f'_m h_e}{E_m}$$

Confined: $$u_e = (22.5\xi_n^2 - 10.4\xi_n + 1.83)\frac{f'_m h_e}{E_m} \quad [39]$$

$$u_s = (1.67\xi_n^2 - 1.64\xi_n - 0.142)\frac{f'_m h_e}{E_m}$$

In these equations, elongation is positive and shortening is negative. It is clear that the tendon force increase due to vertical deformation will increase the axial load ratio. Iteration using Eqns. 38 or 39 is therefore needed to find $\Delta P = \Sigma \Delta P_j$ such that the calculated axial force ratio at nominal flexural strength, ξ_n, injected in the equations on the right hand side in fact corresponds to the calculated tendon force increase on the left hand side of the equations.

The effective total tendon force eccentricity relative to the wall centre line can be evaluated by:

$$e_t = \frac{\Sigma(P_j + \Delta P_j)y_j}{\Sigma(P_j + \Delta P_j)} \qquad \text{where} \quad \Delta P_j = \frac{u_j}{L_j} A_{psj} E_{ps} \quad [40]$$

P_j and ΔP_j are the initial tendon force and tendon force increase of the jth tendon, and y_j is the horizontal location of the jth tendon with respect to the wall centre line taken as positive towards the tension end of the wall. The tendon vertical extension, u_j, is defined in Figure 27 and L_j is the tendon length (approximately the height of the wall h_w, which is significantly longer than h_e for multi-storey building). A_{psj} is the area of the jth tendon and E_{ps} is the elastic modulus of the prestressing steel. It must be ensured that $P_j+\Delta P_j$ does not exceed the tendon yield strength.

Iteration process for calculation of M_n and d_n:

1. Calculate ξ_n using Eqn. 37 using $\Delta P = 0$.
2. Calculate u_e and u_s using Eqns. 38 or 39.
3. Calculate $\Delta P = \Sigma \Delta P_j$ using Eqn. 40.
4. Calculate ξ_n using Eqn. 37 using ΔP from (3).
5. Repeat steps (2) to (4) until convergence of ξ_n.
6. Calculate M_n using Eqn. 31 and d_n using Eqn. 33.

The masonry design codes BS 5628:2005[11] and AS 3700:2001[12] present formulae for calculating the tendon stress increase, but are not applicable for in-plane wall bending because they were developed for out-of-plane response. NZS 3101:2006 recognises that the design tendon force for unbonded tendons will exceed the tendon force following losses. Using the notation presented here, the increase in tendon force is given by:

[11] BS 5628:2005, Part 2: "Code of Practice for use of Masonry. Structural Use of Reinforced and Prestressed Masonry", British Standards Institution, London.

[12] AS 3700-2001, "Masonry Structures", Standard Association of Australia, Homebush, NSW, Australia.

$$\Delta P = A_{ps}\left(70\ \text{MPa} + \frac{f'_m b_w L_w}{100A_{ps}}\right) \quad [41]$$

$$f_{se} = \frac{P}{A_{ps}},\quad f_{ps} \le f_{py} \quad \text{and} \quad f_{ps} \le f_{se} + 400\ \text{MPa} \quad [42]$$

where A_{ps} is the total prestressing tendon area, f_{ps} is the resulting average tendon stress corresponding to $P+\Delta P$, f_{py} is the tendon yield stress, and f_{se} is the tendon stress corresponding to P. This equation seems to provide reasonable results but has not been validated for all wall configurations. It would be prudent to assume a total tendon force increase of ½ - ¾ times the result calculated by Eqn. 41 when the prestressing tendons are approximately evenly distributed along the length of the wall. Eqn. 43 evaluates the resulting tendon eccentricity, e_t, due to the total tendon force increase, assuming that the tendon force increase, ΔP, acts at an eccentricity of $L_w/6$ and that the tendons are evenly distributed across the wall.

$$e_t = \frac{L_w \Delta P}{6(P + \Delta P)} \quad [43]$$

Having calculated ΔP and e_t, the nominal flexural strength, M_n, and corresponding displacement, d_n, can then be evaluated using Eqns. 31 and 33.

4.2.4 Yield Strength

Contrary to reinforced concrete walls, the yield strength for unbonded prestressed walls is typically found at displacements beyond the displacement at nominal flexural strength. Structural testing has consistently shown that the behaviour of unbonded prestressed walls loaded beyond the nominal strength is dominated by rocking as illustrated in Figure 28. Even for walls without specially placed confinement plates, experimental observations consistently demonstrate that the wall is able to support compression strains far beyond 0.003. In Figure 28, the wall has rocked over by a displacement, d_{ty}, corresponding to a rotation θ. At this state, it is assumed that the extreme tendon at the tension side of the wall yields, resulting in a tendon strain increase of:

$$\Delta\varepsilon_{py} = \frac{(f_{py} - f_{ps})}{E_{ps}} \quad [44]$$

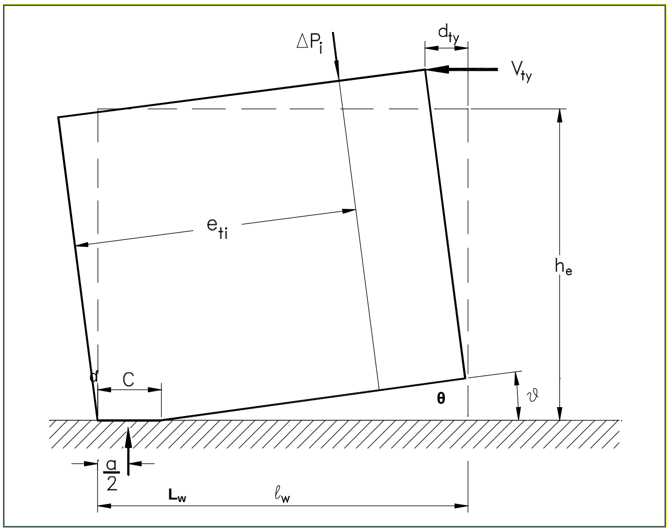

Figure 28: **Rocking Response**

where E_{ps} the modulus of elasticity for the tendon steel, and f_{ps} is taken as the tendon stress in the extreme tendon at nominal strength. If a wall is displaced laterally beyond d_{ty}, some reduction of prestress should be anticipated upon unloading. Notably, this does not mean that wall strength is permanently reduced because the

tendons can be fully activated by subsequent loading excursions. The wall rotation θ can be related to the wall displacement increase at first tendon yield d_{ty} and the tendon strain increase $\Delta\varepsilon_{py}$ in the following way:

$$\theta = \frac{\Delta\varepsilon_{py} h_e}{e_{te} - c} \quad \Rightarrow \quad d_{ty} = \theta h_e = \frac{\Delta\varepsilon_{py} h_e^2}{e_{te} - c} = \frac{f_{py} - f_{ps}}{E_{ps}} \frac{h_e^2}{e_{te} - c} \qquad [45]$$

where $a = \beta c$, and it is assumed β = 0.85 for unconfined masonry and β = 0.96 for confined masonry. In this equation, e_{te} is the eccentricity of the extreme tendon at the wall tension side with respect to the compressive end of the wall. The length of the compression zone, c, is calculated at the nominal strength based on Eqn. 32, thus assuming that the wall rocks about an axis at the distance, c, from the extreme compression fibre in the wall. As d_{ty} is considered as the displacement increment beyond d_n, the stress state in the extreme tendon should rigorously be taken as f_{ps}, however using f_{se} (initial tendon stress in unloaded state) instead of f_{ps} in Eqn. 45 generally results in little error.

Given θ, the force increase in the individual tendons can be calculated as:

$$\Delta P_i = \frac{\theta(e_{tj} - c)}{h_e} E_s A_{psi} = (f_{py} - f_{ps}) A_{psj} \frac{e_{tj} - c}{e_{te} - c} \qquad [46]$$

$$\Delta P_y = \Sigma \Delta P_{tyj} \qquad [47]$$

where e_{tj} is the location of the j'th tendon with respect to the compression end of the wall, A_{psj} is the area of the j'th tendon and ΔP_y is the total tendon force increase above that at M_n. Note that Eqn. 46 assumes linear variation of the tendon force increase with respect to the lateral location of the tendons. The resulting moment increase M_{ty} is then given by:

$$M_{ty} = \sum_{j=1}^{n} \Delta P_{tyj} \left(e_{tj} - \frac{a_y}{2} \right) = \sum_{j=1}^{n} \Delta P_{tyj} e_{tj} - \frac{a_y}{2} \Delta P_y \qquad [48]$$

where n is the total number of tendons along the length of the wall and the compression zone length at first yield may be calculated as:

$$a_y = \frac{P + \Delta P_y + N}{\alpha f'_m b_w} \qquad [49]$$

Finally the yield moment M_y and displacement d_y can be evaluated as:

$$M_y = (N + P + \Delta P)\left(\frac{L_w}{2} - \frac{a_y}{2} \right) + M_{ty} = V_y h_e \qquad [50]$$

$$d_y = d_n + d_{ty} \qquad [51]$$

4.2.5 Flexural Overstrength

The maximum credible strength of an unbonded prestressed wall may be evaluated by assuming that all tendons have reached their yield strength. Consequently, the flexural overstrength, M_o, may be evaluated as:

$$M_o = (N + P_y)\left(\frac{L_w}{2} - \frac{a_o}{2} \right) = V_o h_e \qquad [52]$$

where a_o is the length of the equivalent ultimate compression block and P_y is the total tendon force when all tendons are yielding given by:

$$a_o = \frac{N + P_y}{\alpha f'_m b_w} \quad \text{and } P_y = A_{ps} f_{py} \qquad [53]$$

At this state, it is assumed that the tendon closest to the flexural compression zone has reached its yield stress. The resulting displacement can then be evaluated using the following equation which is similar to Eqn. 45:

$$d_o = d_n + \frac{f_{py} - f_{ps}}{E_{ps}} \frac{h_e^2}{e_{tc} - a_o/\beta} \qquad [54]$$

In this equation e_{tc} is the distance from the compression end of the wall to the closest tendon and f_{ps} is the tendon stress in the same tendon at nominal strength. It is noted that Eqn. 54 is not appropriate if the closest tendon is located within the flexural compression zone, i.e. $e_{tc} < c$, and that if the tendon closest to the compression zone is near to the location of the flexural neutral axis, unrealistically large values of d_o are calculated.

When all tendons are located near the wall centreline, the wall yield strength coincides with the wall overstrength. It can be argued for conservatism that the tendon yield stress, f_{py}, in Eqn. 53 should be replaced with the tendon ultimate strength, f_{pu}, in order to establish the maximum credible wall flexural strength. It is, however, unnecessary to modify Eqn. 54 accordingly because the tendon strain at ultimate strength is of the order of 5% and therefore not attainable in reality for walls of any geometry.

4.2.6 Ultimate Displacement Capacity

The ultimate displacement is limited by the strain capacity of the tendons as well as the crushing strain of the masonry. Generally, the tendon ultimate strain is of the order of 5% which would result in unrealistically high displacement. Consequently, concrete masonry failure is expected. Confinement by the foundation is likely to increase the failure masonry strain beyond 0.003. As the extreme concrete masonry fibres fail, there is a tendency for the compression zone to migrate towards the centre of the wall, reducing the wall strength gradually.

Experiments at the University of Auckland have shown drift ratio capacities of 1% - 2% for prestressed grouted concrete masonry walls of various aspect ratios[9], suggesting high displacement capacity. It is noted that this limit state may occur before tendon yielding, depending on the wall aspect ratio, the prestressing steel area and the initial tendon stress f_{se}.

The drift ratio or the drift angle is defined as the ultimate displacement d_u divided by the effective height:

$$\gamma = \frac{d_u}{h_e} \qquad [55]$$

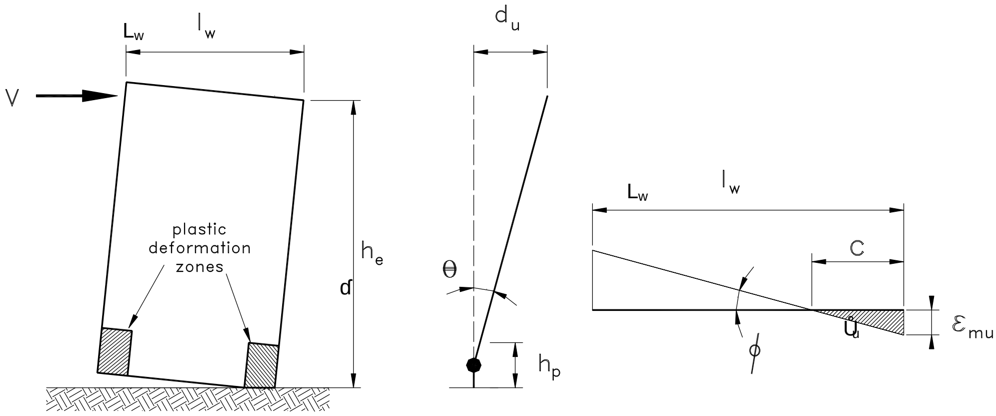

Figure 29: **Vertical Strain Evaluation at Ultimate Displacement Capacity**

Evaluation of the extreme masonry strain at displacements beyond nominal flexural strength necessitates definition of a plastic hinging zone at the bottom of the wall. Assuming that all lateral displacement at the top of the wall is due to rotation, θ, of the plastic hinge as shown in Figure 29, the masonry extreme fibre strain, ε_u , can be related to the wall lateral displacement, d_u:

$$d_u = \theta\left(h_e - \frac{h_p}{2}\right) \quad \text{and} \quad \theta = \phi h_p = \frac{\varepsilon_u}{c} h_p \qquad [56]$$

$$d_u = \frac{h_p\left(h_e - \frac{h_p}{2}\right)}{c}\varepsilon_u \quad \text{where} \quad c = \frac{a}{\beta} = \frac{P + \Delta P + N}{\alpha f'_m b_w \beta} = \frac{\xi_u L_w}{\alpha\beta} \qquad [57]$$

In this equation, ΔP should correspond to the actual tendon stress state at the displacement d_u. It is emphasized that Eqn. 57 is of idealised nature and simply attempts to relate the lateral displacement to the masonry strain state in the compression toe region at the wall state where initiation of strength degradation due to masonry crushing is anticipated to commence. Eqn. 56 assumes that the total rotation occurs at a height of $h_p/2$ above the wall base. This is consistent with the current thinking for plastic hinge zone rotation for reinforced concrete masonry walls[1]. For evaluation of d_u, it is acceptable to interpolate between the axial forces calculated at nominal flexural strength, first tendon yield and overstrength relative to the displacements d_n, d_y and d_o, as applicable (with a maximum of $N+P_y$). The base shear corresponding to d_u can be based on Eqn. 31 using the appropriate axial force or on interpolation between V_f, V_y and V_o with a maximum of V_o.

5.0 Prestressed Masonry Shear Wall

Consider the wall shown in Figure 30(a). It is assumed that the five storey wall is 15 m high, 3.6 m long, 190 mm thick and prestressed with five high strength prestressing strands (A_{psj} = 140 mm^2). Half height 20 series concrete masonry units (100 mm high) are used in the plastic deformation zone; regular 20 series masonry units are used elsewhere. The wall self weight is calculated to be 225 kN and the additional dead load of the floors and roof amounts to 0.5 MPa at the base of the wall.

SOLUTION

Gravity load, N = Wall self weight + additional dead load
= 225 kN + 0.5 x (3600 mm x 190 mm)
= 225 kN + 342 kN
= 567 kN

Calculations are performed on the equivalent single degree of freedom structure shown in Figure 30(b) with an assumed effective height, $h_e = 2/3 \times h_w = 10$ m[**]. The tendons are placed symmetrically about the wall centre line at zero, ±200 mm and ±400 mm eccentricities from the wall centre line (the five strands are represented with one line in Figure 30). In the calculation, the tendon elastic elongation capacity is based on the actual tendon length, approximated as h_w, using an effective tendon elastic modulus of $E_{ps} \times h_e/h_w$. An initial tendon stress of $0.67f_{pu}$ is selected, based on an estimated first tendon yield at a lateral drift of about 1.5% assuming that the wall rocks as a rigid body around the lower corners.

A total prestressing force of A_{ps} x f_{ps} = 700 x 1187 = 831 kN is found, resulting in an initial axial load ratio of ξ = 0.114 (f'_m = 18 MPa).

Confinement plates are imagined embedded in the horizontal bed joints in the wall corners by the base over a height of $2 \times h_p = 2 \times 0.076 \times 10$ m = 1.5 m and K = 1.08 is assumed[9]. The confinement plate length is taken as $2 \times \xi L_w$ or about 800 mm. It is assumed that the height of the plastic hinge zone is $0.076 \times h_e = 0.76$ m (the value of 0.076 was found experimentally by Laursen[9]) and the ultimate flexural strain is 0.008, taken from section 7.4.6.4 or Figure 7.1 of NZS 4230:2004.

** The use of $h_e = 2/3h_w$ is an approximate presentation of moment and shear characteristic in a multi-storey wall with a triangular distribution of lateral loads. For specified lateral loads and storey heights, the relationship may be accurately evaluated from $h_e = \Sigma(h_i V_i)/\Sigma V_i$.

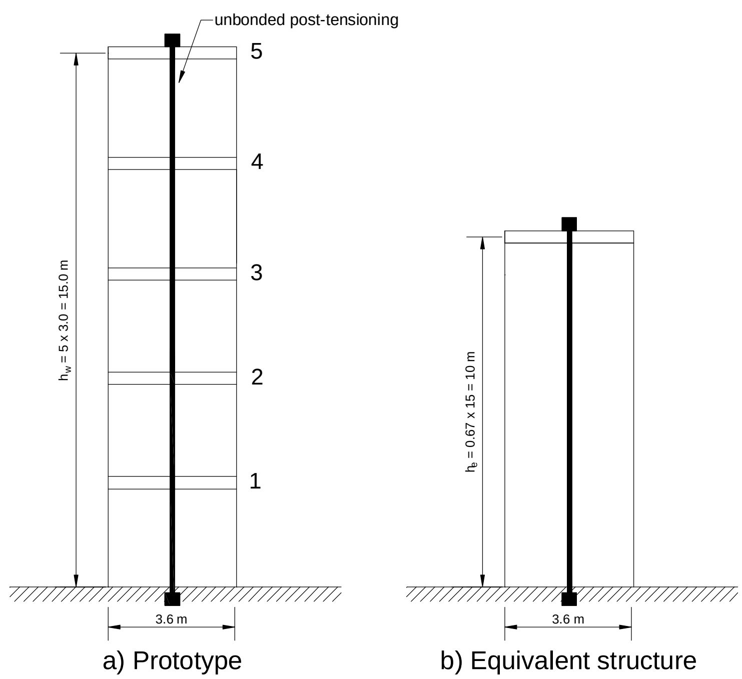

Figure 30: **Post-tensioned concrete masonry cantilever wall**

SOLUTION SUMMARY

Table 12 and Figure 31 present the predicted wall in-plane response with the base shear V, lateral displacement d and tendon force increase ΔP related to the equivalent structure shown in Figure 30(b). Material properties and wall dimensions are specified in Figure 31. Specific details on the calculation example may be found over the page. It is seen in Figure 31 that wall softening initiates between the maximum serviceability moment and the nominal strength limit states. The wall ultimate displacement capacity is reached 83 mm after the nominal strength limit state. The displacement at first tendon yield and wall overstrength is, in this case, only of theoretical interest.

SOLUTION CALCULATIONS

First cracking limit state:

Eqn. 18:
$$M_{cr} = \frac{(567+831)\times 3.6}{6} = 839 \text{ kNm}$$

Eqn. 19:
$$V_{cr} = \frac{839}{10} = 83.9 \text{ kN}$$

Eqn. 20:
$$d_{cr} = \frac{2}{3}\times\frac{(567+831)\times 10^2}{14400\times 3.6^2\times 0.19} + \frac{2}{5}\times\frac{(1+0.2)\times(567+831)}{14400\times 0.19} = 0.0029 \text{ m}$$

Maximum serviceability moment:

Eqn. 24:
$$M_e = 2.04\times\left(0.5-1.21\times\frac{2.04}{18}\right)\times 3.6^2\times 0.19 = 1820 \text{ kNm}$$

$$V_e = \frac{1820}{10} = 182 \text{ kN}$$

Eqn. 28:
$$d_e = \left(0.3-0.029\times\frac{2.04}{18}\right)\times\frac{18\times 10^2}{14400\times 3.6} + \frac{12}{5}\times\frac{(1+0.2)\times 10}{14400\times 3.6\times 0.19}\times\frac{182}{1000} = 0.0108 \text{ m}$$

Table 12: Predicted force and displacement

	First cracking	Maximum serviceability moment	Nominal strength	Ultimate displacement capacity	First tendon yield	Wall over-strength	
V	83.9	182	227	242	248	253	**kN**
d	2.9	10.8	41.2	124	158	310	**mm**
ΔP	0	0	34	109	140	231	**kN**

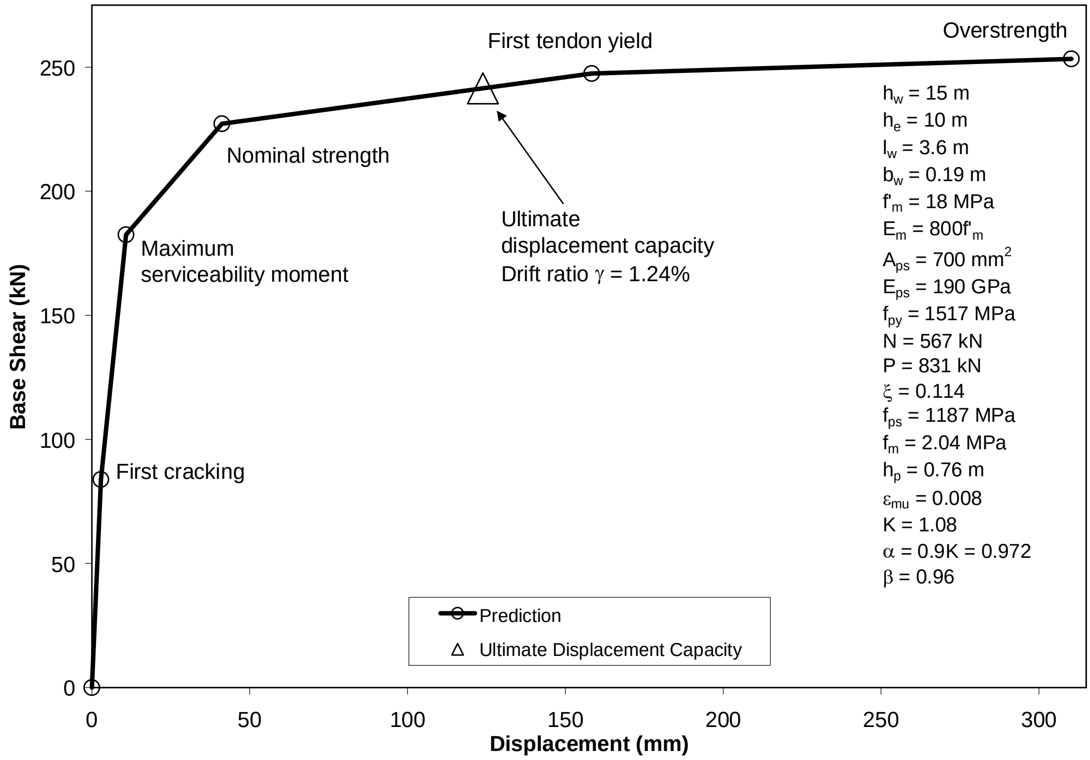

Figure 31: **Predicted in-plane response**

Nominal strength:

First iteration using $\xi_n = 0.114$:

Eqn. 39: $u_e = 0.0117$ m and $u_s = -0.00384$ m

Eqn. 40: $\Delta P_1 = 10.1$ kN, $\Delta P_2 = 8.5$ kN, $\Delta P_3 = 7.0$ kN, $\Delta P_4 = 5.5$ kN, $\Delta P_5 = 3.9$ kN

and $\Delta P = 35.0$ kN, $e_t = 0.004$ m → $\xi_n = 0.116$

Second iteration using $\xi_n = 0.116$:

Eqn. 39: $u_e = 0.0115$ m and $u_s = -0.00387$ m

Eqn. 40: $\Delta P_1 = 9.8$ kN, $\Delta P_2 = 8.3$ kN, $\Delta P_3 = 6.8$ kN, $\Delta P_4 = 5.3$ kN, $\Delta P_5 = 3.8$ kN

and $\Delta P = 34.0$ kN, $e_t = 0.004$ m → $\xi_n = 0.116$ (therefore OK)

Eqn. 32: $$a = \frac{831 + 34 + 567}{0.972 \times 18 \times 0.19} = 0.431\,\text{m}$$

Eqn. 31: $$M_n = (831 + 34) \times \left(\frac{3.6}{2} + 0.004 - \frac{0.431}{2}\right) + 567 \times \left(\frac{3.6}{2} - \frac{0.431}{2}\right) = 2272\,\text{kNm}$$

$$V_f = \frac{2272}{10} = 227\,\text{kN}$$

Eqn. 33: $$d_n = (7.63 \times 0.116^2 - 5.40 \times 0.116 + 1.69) \times \frac{18 \times 10^2}{14400 \times 3.6} + \frac{12}{5} \times \frac{(1 + 0.2) \times 10}{14400 \times 3.6 \times 0.19} \times \frac{227}{1000}$$

$$= 0.0412\,\text{m}$$

Stress in tendon furthest away from compression end of wall:

$$f_{ps1} = \frac{(P_1 + \Delta P_1)}{A_{ps1}} = \frac{831/5 + 9.8}{140} = 1257\,\text{MPa}$$

Stress in tendon closest to compression end of wall:

$$f_{ps5} = \frac{(P_5 + \Delta P_5)}{A_{ps5}} = \frac{831/5 + 3.8}{140} = 1214\,\text{MPa}$$

First tendon yield:

c = a/ɓ= 0.431/0.96 = 0.449 m (ɓ = 0.96 for confined masonry)

Eqn. 45: $$d_{ty} = \frac{1517 - 1257}{190000 \times 10/15} \times \frac{10^2}{3.6/2 + 0.4 - 0.449} = 0.1172\,\text{m}$$

where h_e/h_w = 10/15 modifies E_{ps} to reflect the actual tendon length.

Eqn. 46: $$\varphi P_{ty1} = (1517 - 1257) \times 140 \cdot \frac{3.6/2 + 0.4 - 0.449}{3.6/2 + 0.4 - 0.449} = 36.4\,\text{kN}$$

$$\Delta P_{ty2} = 32.2\,\text{kN}$$

$$\Delta P_{ty3} = 28.1\,\text{kN}$$

$$\Delta P_{ty4} = 23.9\,\text{kN}$$

$$\Delta P_{ty5} = 19.8\,\text{kN}$$

Eqn. 47: $$\Delta P_y = 140.4\,\text{kN}$$

Eqn. 49: $$a_y = \frac{831 + 140 + 567}{0.972 \times 18 \times 0.19} = 0.463\,\text{m}$$

Eqn. 48: $$M_{ty} = 36.4 \times \left(\frac{3.6}{2} + 0.4\right) .....19.8 \times \left(\frac{3.6}{2} - 0.4\right) - \frac{0.463}{2} \times 140 = 229\,\text{kNm}$$

Eqn. 50: $$M_y = (831 + 34 + 567) \times \left(\frac{3.6}{2} - \frac{0.463}{2}\right) + 229 = 2475 \text{ kNm}$$

$$V_y = \frac{2475}{10} = 248 \text{ kN}$$

Eqn. 51: $$d_y = 0.041 + 0.1172 = 0.158 \text{ m}$$

Overstrength:

Eqn. 53: $$P_y = 5 \times 140 \times 1517 = 1062 \text{ kN}$$

$$a_o = \frac{1062 + 567}{0.972 \times 18 \times 0.19} = 0.490 \text{ m}$$

Eqn. 52: $$M_o = (1062 + 567) \times \left(\frac{3.6}{2} - \frac{0.490}{2}\right) = 2533 \text{ kNm}$$

$$V_o = \frac{2533}{10} = 253 \text{ kN}$$

Eqn. 54: $$d_o = 0.0412 + \frac{1517 - 1214}{190000 \times 10/15} \times \frac{10^2}{3.6/2 - 0.4 - 0.490/0.96} = 0.310 \text{ m}$$

Ultimate displacement capacity:

First iteration:

Assume: $$c = \frac{\frac{1}{2}\left(a + a_y\right)}{\beta} = \frac{\frac{1}{2} \times (0.431 + 0.463)}{0.96} = 0.466 \text{ m}$$

Eqn. 57: $$d_u = \frac{0.76 \times \left(10 - \frac{0.76}{2}\right)}{0.466} \times 0.008 = 0.126 \text{ m}$$

Second iteration:

Using d_u found in Eqn. 57, interpolate between a and a_y to find c.

$$c = \frac{a + \frac{a_y - a}{d_y - d_n}(d_u - d_n)}{ƃ} = \frac{0.431 + \frac{0.463 - 0.431}{0.158 - 0.041} \times (0.126 - 0.041)}{0.96} = 0.473 \text{ kN}$$

Eqn. 57: $$d_u = \frac{0.76 \times \left(10 - \frac{0.76}{2}\right)}{0.473} \times 0.008 = 0.124 \text{ m} \rightarrow \text{OK}$$

The wall strength at d_u is found by interpolation between nominal strength and first tendon yield limit states with respect to displacement:

$$V_u = V_f + \frac{V_y - V_f}{d_y - d_n}(d_u - d_n) = 227 + \frac{248 - 227}{0.158 - 0.041} \times (0.124 - 0.041) = 242 \text{ kN}$$